Particle image velocimetry measurement of an instability wave over a porous wall in a duct with flow

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ABSTRACT

The flow in a rectangular channel lined with a porous material and acoustically excited with an upstream loudspeaker has been investigated using particle image velocimetry. The measurements are phase-locked to the loudspeaker signal so that the phase-averaged velocity in the lined section is obtained during an excitation period. Most features of the phase-averaged velocity field in the lined section are found to be well described from the sum of three single duct modes: the hydrodynamic instability wave, a standing wave and an acoustic wave. The hydrodynamic instability wave travels at half the mean flow velocity, and its structure shows differences to the case of a locally reacting liner. The relative phase lag between the hydrodynamic and acoustic waves at the liner end dictates the interference between both waves, giving rise to the oscillations of the acoustical transmission coefficient as a function of the frequency. A detachment of the instability wave from the porous wall is observed in the vicinity of the liner downstream edge, together with the separation of the mean vorticity core.

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1. Introduction

In many industrial applications where noise propagates through pipes and ducts, acoustic treatments are used to reduce the sound transmitted to the surroundings. To diminish the energy losses in the mean flow, the acoustic treatments are located in the walls of flow ducts and they are called acoustic liners. There exist two families of liners. The first family includes the locally reacting liners, i.e. liners that are locally equivalent to an acoustic impedance. They are efficient tonal noise attenuators and are widely used in aero-engines [1]. The second family includes porous liners, which are non-locally reacting. They are effective broadband noise attenuators [2] and are commonly used in the air conditioning and ventilation systems, as well as in exhaust systems for gas turbines or IC-engines.

There is an important effect of flow on the performance of locally reacting and porous liners. The interaction of the boundary layer with the lined wall changes the modal structure existing in absence of flow [3–6]. Rienstra [5] showed theoretically that there can exist "strange modes" in a flow pipe with a locally reacting liner. These modes can be divided into acoustic and hydrodynamic surface modes, and their number and stability character depends on the liner impedance, as well as the flow Mach number. It was also shown that for a certain range of impedances, at least one hydrodynamic mode was unstable.

Previous experimental investigations have revealed a peak of the transmission coefficient around the resonance frequency of locally reacting liners, with the peak frequency and level depending on the flow Mach number [7–9]. Alongside

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with the transmission coefficient peak, a static pressure drop along the lined section was detected. These results were attributed to the presence of an unstable hydrodynamic mode over the liner. Marx et al. [10] studied experimentally a low resistance liner using particle image velocimetry (PIV) and laser Doppler velocimetry (LDV). A linear, convectively unstable mode was directly observed in the lined section, and its wavelength, phase velocity, amplification and wall-normal eigenfunctions were obtained.

The case of flow ducts lined with a porous material is also of fundamental and practical interest. A porous liner is not locally reacting, and the acoustic fields inside and outside the porous material cannot be decoupled. They both need to be calculated simultaneously [11,12]. Aurégan and Singh [13] investigated experimentally the acoustic properties of a homogeneous porous liner in a flow duct. They observed oscillations of the transmission coefficient in a wide range of frequencies. The transmission coefficient was decomposed into a ‘hydrodynamic’ contribution, responsible for the oscillations, and an ‘acoustic’ contribution, similar to the transmission coefficient without flow. These two contributions were associated with an hydrodynamic wave and an acoustic wave, respectively. The oscillations of the transmission coefficient were then hypothesised to be due to the interference between the hydrodynamic and acoustic waves at the downstream liner section. This picture differs from the locally reacting liner case, where acoustic resonance plays a crucial role in the instability mechanism.

The current work aims at further investigating the unstable mode over a porous liner with grazing flow. PIV has been used to measure the velocity field in the lined section. This way the complete spatial structure of the waves present can be observed. This will lead to a better modelling, which will improve its prediction and prevention in applications.

A preliminary version of this study was presented as a conference paper [14]. The current version contains additional results and extended discussions. In particular, proper orthogonal decomposition has been used to ‘clean’ the measured velocity fields, an analysis of the mean vorticity has highlighted new features of the mean velocity field, the velocity signals in time have been analysed, further results and discussion concerning the detachment of the instability wave have been included, and finally, the impact of the instability wave to the turbulent field has been assessed. For completeness, some figures from [14] are presented in the current paper.

In Section 2 the experimental rig and the measuring technique are presented, together with results from microphone measurements. Section 3 describes the phase-locking technique, and the procedure followed to split the velocity field into mean, phase-averaged and turbulent components. In Section 4 the results are presented and discussed.

2. Experimental setup

2.1. Duct rig

The rig used consists of a rectangular duct composed of an upstream segment of dimensions 8 cm (width) × 3 cm (height) and a downstream segment containing the test section, with dimensions 8 cm (width) × 2 cm (height). Both are united by a 0.2 m long convergent. The downstream duct segment is 0.6 m long, and has an anechoic termination. The lined wall is located at the centre of the test section. It spans the entire channel width and has a length of 8 cm. The test section has two rectangular windows, one on the opposite wall to the liner and another at a side wall. The former lets the laser sheet pass through, illuminating the entire lined section, and the latter allows the light reflected to reach the camera. The flow through the duct was generated with a fan providing a mean centreline velocity up to about 100 m/s. The incoming sound is generated by a loudspeaker fixed in the upstream duct segment, at 1.5 m from the test section. The sound level at the entrance of the test section was 143 dB, in order to maximise the signal-to-noise ratio. The same rig was used by Marx et al. [10], and it is sketched in Fig. 1.

2.2. Porous liner

The porous material consists of a rigid metallic foam (RECEMAT, NC4753.05 nickel-chromium alloy), which is the same used by [13]. The parameters of the porous material in the fluid equivalent model [15] were determined to be: porosity \( \Phi = 0.99 \), tortuosity \( \alpha_m = 1.17 \), viscous length \( \Lambda = 1 \times 10^{-4} \text{m} \), thermal length \( \Lambda' = 2.4 \times 10^{-4} \text{m} \), and resistivity \( \sigma = 6.9 \times 10^3 \text{kg} \text{m}^{-1} \text{s}^{-1} \). The 25 mm depth cavity was filled with five plates of thickness 5 mm each, rigidly assembled to

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prevent any empty spaces between the plates and between the porous material and the cavity walls.

Four microphones flush to the wall were used to measure the upstream and downstream plane wave amplitudes, and the transmission and reflection coefficients. The microphones were B&K 4938 (1/4 in), and the preamplifier was a B&K 2670 with Nexus. The calibration procedure consisted of an absolute calibration, needed to determine the sound level, and a relative calibration between one of the microphones and each one of the other three. The absolute calibration was performed using a 1 kHz tone generator at 90 dB. In the relative calibration the two microphones were fixed opposite to each other in a closed, symmetric resonant cavity, excited with a loudspeaker. The transfer function was then determined in the frequency range of interest.

During the measurements a pair of microphones was mounted upstream of the test section, and another pair was mounted downstream (see Fig. 1). The distance between the microphones in each pair is 8.5 cm. The upstream microphone of the upstream pair is located at \( x = -17 \) cm, and the upstream microphone of the downstream pair at \( x = 17 \) cm (axis origin is on the liner upstream edge). On the setup there is only an upstream source. The upstream transmission and reflection coefficients have been determined by neglecting the small reflection from the “anechoic” termination (the reflection coefficients from the anechoic end at 1220 Hz and 1500 Hz, i.e the frequencies corresponding to the PIV measurements, are 0.13 and 0.19, respectively). The transmission coefficient is shown in Fig. 2. The grey band covers the region around 2 kHz, where the two-microphone method failed due to the distance between the microphones being close to half the acoustic wavelength. The ripples detected by Aurégan and Singh [13] are also observed.

2.3. PIV

A Dantec PIV system has been used. It consists of a Litron laser pulse generator of 532 nm, synchronised with a high-speed FlowSenseEO 29 M camera, of 6600 × 4400 pixels. The time between images was 1.5 μs, and the time between image pairs was 0.4 s. An adaptive correlation algorithm provided by the Dantec software was used, which allowed to increase the cross-correlation coefficient and reach a window size of 32 × 32 pixels. To increase the resolution, an overlap of 50% was used. The PIV plane is shown in Fig. 3. To eliminate errors from merging adjacent vector fields, the entire lined section was captured from a single snapshot. This caused the aspect ratio of the PIV plane to be large, and the number of pixels along the channel height was limited to 610. Finally, the PIV plane contained 161 (streamwise) × 32 (wall-normal) vectors.

![Fig. 2. Measured transmission coefficient at \( M = 0.25 \) (---) and \( M = 0 \) (—) [14].](image)

3. Velocity fields

PIV measurements were performed for two excitation frequencies: 1500 Hz and 1220 Hz. These frequencies correspond to a peak (1500 Hz) and a trough (1220 Hz) of the transmission coefficient (Fig. 2). The measurements were phase-locked to the excitation signal, i.e. the laser and camera signals were synchronised with the loudspeaker so that the image pairs could be taken at fixed phases with respect to the loudspeaker driving signal. The excitation period was split in 10 uniformly distributed intervals, as shown in Fig. 4.

In order to average out the turbulent component, \(N = 120\) vector fields were measured for each of the \(N_p = 10\) phases. The streamwise (wall-normal) velocity measured at the spatial location \(x, y\), corresponding to the \(i\)th image pair at phase \(p\), is \(u(x, y, i, p)\) (\(v(x, y, i, p)\)). The Reynolds number of the mean flow in the duct, based on the bulk velocity, is 110,000 and the flow is fully turbulent. Using a similar notation as Marx et al. [10] the \(i\)th instantaneous streamwise velocity field at phase \(p\) is

\[
\begin{align*}
    u(x, y, i, p) &= U(x, y) + u'(x, y, p) + u_i(x, y, i, p), \\
    &\quad i = 1, \ldots, N \quad \text{and} \quad p = 1, \ldots, N_p,
\end{align*}
\]

Fig. 4. Loudspeaker signal and camera capturing instants at phases \(p = 1, \ldots, 10\) [14].

Fig. 5. (a) Squared singular values (in \(m^2/s^2\)). (b) Phase-averaged velocity along \(y = 3.9\) mm at \(p = 3\) for the 1500 Hz excitation. (c) Phase-averaged velocity along \(y = 3.9\) mm at \(p = 3\) for the 1220 Hz excitation.

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where \( U \) is the mean field, \( u' \) is the phase-averaged velocity (periodic fluctuation) and \( u_t \) is the turbulent velocity (stochastic fluctuation). The sum of the mean and the phase-averaged velocity is the average of all the velocity fields at a given phase. The mean field is obtained from averaging the entire set of velocity fields for all phases. The phase-averaged velocity is obtained from averaging all velocity fields at one particular phase and then subtracting the mean field. Finally, the turbulent velocity \( u_t \) is readily obtained from Eq. (1) once the phase-averaged and mean components are known.

The error of the mean and phase-averaged velocity fields have been estimated from

**Fig. 6.** Obtention of the mean, phase-averaged and turbulent velocity components from the (phase-locked) measured velocity fields.

**Fig. 7.** Mean velocity profiles at \( x = -5 \text{ mm} \) (×) and \( x = 75 \text{ mm} \) (○), for the 1500 Hz excitation [14].
σ\,(x,\,y) \approx \frac{u_{rms}(x,\,y)}{\sqrt{NN_{p}}},

(2)

\sigma'(x,\,y,\,p) \approx \frac{\sum_{i=1}^{N}u_{i}^{2}(x,\,y,\,i,\,p)}{N},

(3)

which is the error induced from estimating a mean from a finite sample, and increases with the standard deviation of the sample. The direct PIV measurement error is much smaller in all cases and has been neglected.

We focus now on the phase-averaged velocity. In order to diminish the error of \(u', v'\) a proper orthogonal decomposition...
(POD) [16] has been applied to the phase-averaged velocity fields. POD provides reduced order approximations of a velocity flow field, which are optimum with respect to the total unsteady kinetic energy. The optimisation problem is equivalent to an eigenvalue problem, whose solution consists of (1) a set of (orthogonal) eigenmodes forming a complete basis in which the flow field can be expanded and (2) the eigenvalues associated to each mode, which are a measure of its contribution to the total kinetic energy. In particular, the method of snapshots [17] has been used, which is more efficient than classical POD in the present case where the number of snapshots is much smaller than the number of velocity vectors per snapshot. The problem in this case is equivalent to a singular value decomposition of a data matrix. The result is an expansion of the measured velocity field in a number of reduced order velocity fields. The weight of each mode in the expansion is the corresponding singular value, which have units of velocity (m/s). The square of the singular values corresponds to the kinetic energy linked to each singular mode. Fig. 5(a) shows the squared singular values at both excitation frequencies. At 1500 Hz, the first two singular modes contain 77% of the total energy. For the 1220 Hz excitation the energy fall-off is milder and more modes are needed to keep the same fraction of kinetic energy. The first two singular modes contain barely 50% of the kinetic energy linked to each singular mode. Fig. 5(b,c) show the measured and low-rank approximation of the velocity field along $y = 3.9$ m at the phase $p = 3$, for 1500 Hz (b) and 1220 Hz (c). The low-rank curves contain less random fluctuations than the measured ones. The low-rank approximation of the phase-averaged velocity fields has been used instead of the original.

In the linear regime the system is fully determined by the fundamental harmonic of the Fourier time series:

$$u'(x, y, p) \approx \text{Re}(\hat{u}(x, y)\exp(j2\pi(p - 1)/N_p)), \quad p = 1, \ldots, N_p,$$

$$\hat{u}(x, y) = \frac{2}{T} \sum_{p=1}^{N_p} u'(x, y, p)\exp(-j2\pi(p - 1)/N_p) = \text{abs}(\hat{u}(x, y))\exp(j\phi^u(x, y)),$$

where $\hat{u}$ is the fundamental Fourier mode defined by its amplitude $\text{abs}(\hat{u})$ and phase $\phi^u$. As will be shown, the contribution from the second Fourier harmonic (at twice the excitation frequency) is important in a certain flow field region. The corresponding Fourier mode $\hat{u}_2(x, y)$ is obtained analogously from the Fourier time series. It has been verified that the contribution from higher order modes is small.
The summary of the steps followed to obtain the mean, phase-averaged and turbulent velocity components from the measured velocity fields is shown in Fig. 6.

All the above expressions apply also to the wall-normal velocity field by changing “u” to “v”.

4. Results

4.1. Mean velocity field

Mean velocity profiles in the upstream rigid wall section and the lined section are shown in Fig. 7. The boundary layer thickens along the porous wall, and the centreline velocity increases. A similar phenomenon happens for a channel with a roughness patch [18]. Mass conservation implies an acceleration of the flow in the central region of the channel together with a wall-normal displacement of the streamlines away from the lined wall. Fig. 8 shows the wall-normal velocity contour. It reveals a positive wall-normal velocity region in the vicinity of the liner end.

Another relevant aspect of the mean field concerns vorticity. The mean vorticity profiles in an upstream rigid wall section ($x = -1.5\, \text{mm}$) and a lined section close to the liner end ($x = 75\, \text{mm}$) are shown in Fig. 9. While in the rigid section vorticity peaks at the wall, at $x = 75\, \text{mm}$ it peaks at a certain distance from the porous wall. It appears here that the vorticity peak in the downstream region is shifted away from the porous wall by the mean wall-normal velocity (Fig. 8). As the vorticity peak is not sharp but shows a smooth distribution, it is referred to as the vorticity core. The inhomogeneities of the wall-normal velocity field and the detachment of the vorticity core were not observed by Marx et al. [10] for a locally reacting liner. These features indicate that the boundary layer is significantly perturbed by the porous wall and suggest the presence of flow inside the porous material.

4.2. Phase-averaged velocity field

Examples of a phase-averaged streamwise velocity contour and a turbulent (instantaneous) streamwise velocity contour,

<table>
<thead>
<tr>
<th>Wave properties</th>
<th>1220 Hz</th>
<th>1500 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_u$ (m$^{-1}$)</td>
<td>$153 \pm 2$</td>
<td>$202 \pm 2$</td>
</tr>
<tr>
<td>$k_w$ (m$^{-1}$)</td>
<td>$17.5 \pm 5$</td>
<td>$20.2 \pm 3$</td>
</tr>
<tr>
<td>$k_{uw}$ (m$^{-1}$)</td>
<td>$27 \pm 2$</td>
<td>$47 \pm 6$</td>
</tr>
<tr>
<td>$k_{uw}$ (m$^{-1}$)</td>
<td>$-16 \pm 6$</td>
<td>$-21 \pm 3$</td>
</tr>
<tr>
<td>$\lambda_h$ (m)</td>
<td>$0.041$</td>
<td>$0.031$</td>
</tr>
<tr>
<td>$\lambda_v$ (m)</td>
<td>$0.36$</td>
<td>$0.31$</td>
</tr>
<tr>
<td>$c_s$ (m/s)</td>
<td>50.4</td>
<td>47.1</td>
</tr>
<tr>
<td>$c_a$ (m/s)</td>
<td>439</td>
<td>465</td>
</tr>
<tr>
<td>$\phi^u - \phi^V$ (rad)</td>
<td>$0.64\pi$</td>
<td>$0.63\pi$</td>
</tr>
<tr>
<td>$\theta^u - \theta^V$ (rad)</td>
<td>$-0.62\pi$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1

Properties of the instability, standing and acoustic waves (confidence intervals correspond to $2\sigma$).

Fig. 12. Real and imaginary components of the wavenumbers corresponding to the instability wave (●), the acoustic wave (▼) and the trace of the standing wave with negative phase velocity (×), for the 1500 Hz excitation [14].
both corresponding to the phase \( p = 3 \), are shown in Figs. 10(a) and (b), respectively. The difference in magnitude between the turbulent and phase-averaged velocity is appreciated. A set of phase-averaged velocity contours along an entire period at both excitation frequencies are included in the Appendix (Figs. A1 and A2, respectively). A wave-like pattern convecting and amplified downstream until the vicinity of the liner downstream edge is observed. The pattern is similar to the one observed by Marx et al. for a locally reacting liner, with maximum intensity close to the liner and monotonically decreasing away from the wall. These attributes are consistent with a convectively unstable wave close to the surface. The presence of a counter flow between the wave fronts and the lined wall in the vicinity of the liner downstream edge is also appreciated in the streamwise velocity contours, which is not observed in the wall-normal velocity. This feature is discussed in Section 4.2.3. The streamwise velocity also has high levels close to the opposite rigid wall, associated to the incoming sound waves. The contours of the fundamental Fourier mode (Eq. (5)) are also included in the Appendix (Figs. A3 and A4). Except for the streamwise velocity at 1220 Hz, the maximum values of the velocity are attained downstream, in the vicinity of the porous wall. The phase contours resemble a saw-tooth pattern in the vicinity of the lined wall, consistent with a downstream propagating wave (discontinuities introduced by the principal branch of the inverse tangent function).

Figs. 11(a,b) show the velocity signal along a period at the single point \((x, y) = (75, 5)\ mm\). For the 1500 Hz excitation the fundamental Fourier harmonic matches well the measured signals. This indicates the system response is close to linear. Similar trends are found elsewhere in the lined section, with the exception of a small region (Section 4.2.3). At 1220 Hz the same is true for the wall-normal velocity, but for the streamwise velocity the match is poorer. It is precisely for the streamwise velocity at 1220 Hz where the phase-averaged velocity is weaker with respect to the turbulent velocity, and the error of the phase-averaged velocity is larger.

### 4.2.1. Decomposition into hydrodynamic, standing and acoustic waves

In order to find the dominant duct modes in the lined section, a Fourier transform in the streamwise direction is performed. The resulting \( k \)-spectrum reveals three important contributions to the phase-averaged velocity field. By order of magnitude their real parts are: (1) \( \omega/c_b \), (2) \(-\omega/c_b \), and (3) \( \omega/c_a \). Due to the limited spatial accuracy, only a rough estimation of

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**Fig. 13.** Streamwise (a) and wall-normal (b) velocity eigenfunctions corresponding to the instability wave (a), the standing wave (b) and the acoustic wave (c), for the 1500 Hz excitation. Parabolic best-fits are also shown (- - -) [14].

**Fig. 14.** Phase lags of the streamwise (a) and wall-normal (b) velocity components corresponding to the instability wave (a), the standing wave (b) and acoustic wave (c), for the 1500 Hz excitation. The linear regressions are also shown (- - -) [14].

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The peak wavenumbers can be obtained this way (more precise values will be obtained from a best fit with the duct wave modes). The first contribution corresponds to an amplified wave convecting downstream. In agreement with previous measurements with a porous liner [13] and a locally reacting liner [10], it propagates at approximately half the mean flow velocity \( \approx \frac{c_U}{2} \), where \( U_{\text{bulk}} = \frac{1}{LH} \int U(x, y) \, dx \, dy \). This wave is the result from an hydrodynamic instability excited by the incoming sound. The measured value of \( c_0 \) coincides with the linear stability theory of a free vortex sheet [19]. The third contribution is associated to an attenuated wave propagating downstream at a velocity consistent with an acoustic wave \( \sim 340 \, \text{m/s} \). There must exist other acoustic wave modes but their amplitude is supposed to be too small to measure them. The second contribution is associated to a perturbation with the same wavelength as the instability wave, but with a phase velocity of opposite sign. We interpret it as the manifestation of a standing wave with the same wavelength as the instability wave, and whose origin is unclear. Such a standing wave cannot be explained in the linear regime, and it may be generated by a nonlinear effect. A hypothesis of such an effect are wall-normal oscillations of the boundary layer in its interaction with the porous wall, induced by the instability wave. The phase-averaged velocity in the lined section is then modelled as the superposition of the hydrodynamic instability wave \( \hat{u}_h \), the standing wave \( \hat{u}_s \) and the acoustic wave \( \hat{u}_a \): \[
\hat{u}(x, y) = \hat{u}_h(x, y) + \hat{u}_s(x, y) + \hat{u}_a(x, y) = A_h^u(y) \exp(k_h(x - L) - jk_{bi}y + j\theta_h^u) + A_s^u(y) \exp(k_s(x - L) + j\theta_s^u + jk_{bi}y) + A_a^u(y) \exp(k_a(x - L) - jk_{ar}y + j\theta_a^u),
\]
where \( A_h^u, A_s^u, A_a^u \) are amplitude eigenfunctions, \( k_h = k_{bi} + jk_{bi}, \) \( k_s = k_{ar} + jk_{ar}, \) \( k_a = k_{ar} + jk_{ar} \) are the wavenumbers, and \( \theta_h^u, \theta_s^u, \theta_a^u \) are the phase lags. The amplitude eigenfunctions correspond to the velocity amplitudes at the liner end (\( x = L \)).
wavelength and the phase velocity are directly related to the real component of the wavenumbers by \( \lambda = \frac{2\pi}{k} \) and \( c = 2\pi f / k \), respectively. The decomposition of the wall-normal velocity is analogous, with the same wavenumbers. The model is corrected for the saturation of the hydrodynamic wave simply by the cut-off of the amplification, which happens at about \( x = 63 \) mm for the streamwise velocity and \( x = 70 \) mm for the wall-normal velocity. The amplitude eigenfunctions, wavenumbers and phase lags are deduced from a least-squares fit with the measurements from \( x = 0 \) mm until saturation, for fixed values of \( y \). At 1220 Hz the standing wave contribution is too small to be determined consistently across the channel, therefore only the hydrodynamic and the acoustic wave modes have been considered. The resulting model values at 1500 Hz and 1220 Hz are shown in Table 1.
Theoretically many acoustic modes are present in the lined section, and reducing them to a single acoustic mode could be seen as a strong simplification. However, the least attenuated mode becomes dominant shortly after the wave enters the lined region, and shortly before it reaches the liner end. It is only in the immediate vicinity of the liner edges where higher order modes are important. Indeed, it is found that removing the vicinity of the liner upstream edge from the fit in Eq. (6) does not change the resulting wavenumbers and amplitudes significantly, and therefore the contribution from high order modes is not important overall. The reduction of the acoustic field to a single wave mode was implicitly done by Aurégan and Singh [13] to model the transmission coefficient.

The wavenumbers obtained from the model fit at 1500 Hz are shown in Fig. 12. The mode with negative phase velocity is a trace of the standing wave, which is composed of positive and negative phase velocity components. The wavenumber values obtained at both excitation frequencies are included in Table 1. An additional source of error in the determination of the acoustic mode wavenumber is due to the small lined section length with respect to the acoustic wavelength (ratio equal to 0.27 at 1500 Hz and 0.2 at 1220 Hz).

The amplitude eigenfunctions at 1500 Hz are shown in Figs. 13(a–c). The fitted curves are parabolas obeying the boundary condition (for inviscid flow) at the opposite rigid wall (\( \frac{\partial p}{\partial y} = 0 \)). The amplitude of both velocity components of the hydrodynamic instability wave have similar levels across the channel, and peak at the porous wall (Fig. 13(a)). This is the behaviour of the hydrodynamic mode predicted by a linear solver with uniform flow. The streamwise velocity eigenfunction is largely different from the one corresponding to a locally reacting liner [10]. The latter peaks sharply at the lined wall, and it has lower levels away from the liner. The eigenfunctions corresponding to the standing wave are similar to the instability wave: they peak at the porous wall and both velocity components have similar levels (Fig. 13(b)). Regarding the acoustic wave, within \( 5 < y < 15 \text{ mm} \) the amplitude shows a consistent increase towards the opposite rigid wall (Fig. 13(c)). Such behaviour coincides with the eigenfunction of the least attenuated acoustic mode corresponding to uniform flow.

The phase lags show consistent linear trends across the duct (Figs. 14(a–c)). Such linear variations were also detected for a locally reacting liner [10]. A linear fit to the optimum curves is used. The phase lag difference between \( U_h \) and \( V_h \) is slightly larger than \( \pi/2 \) rad (Fig. 14(a)), the value predicted by the linear stability theory of a free vortex sheet (Kelvin–Helmholtz instability). For a locally reacting liner it was found to be 0.15\( \pi \) rad [10]. The phase lag difference between the two velocity components for the standing wave is close to the instability wave, but with negative sign. The phase lag of the acoustic wave is close to the one of the instability wave, indicating that both waves are in phase at \( x=0 \). This is expected as it is the incoming acoustic wave that excites the instability wave at the liner entrance.

The three wave modes along the lined section are shown in Fig. 15. The ‘measured’ curves are obtained from subtracting the other two modes to the measured Fourier mode. The velocity amplitude of the hydrodynamic wave follows accurately an exponential growth (Fig. 15(a)), and its phase decreases linearly (Fig. 15(d)). The match in the phase in the middle and downstream liner regions is remarkable, even downstream of saturation. There the velocity wave fronts are not exponentially amplified, but they still travel at the expected phase velocity, which is well captured by the model and determines the phase of the wave. The real and imaginary components of the standing wave are shown in Fig. 15(b) and (e), respectively. The imaginary component is weaker than the real component. Again, the model describes well the experimental trend downstream of saturation. The streamwise velocity of the acoustic wave mode is plotted instead of the wall-normal velocity because the latter is very weak. The acoustic wave is attenuated downstream (Fig. 15(e)) and its phase decreases linearly (Fig. 15(f)), at a much lower rate than the instability wave. Due to its weakness with respect to the instability wave, the determination of the standing and acoustic wave is less accurate.

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For the 1500 Hz excitation the model (Eq. (6)) matches well the measured velocity amplitude (Fig. 16(a)) and phase (Fig. 16(c)), even downstream of saturation. The amplitude of the acoustic wave is significantly larger than the other two modes in the vicinity of the liner upstream edge, and there the amplitude and the phase of the total velocity approach the values of the acoustic wave mode. From about the middle section of the liner the instability mode becomes dominant. The contribution of the hydrodynamic and acoustic modes to the streamwise velocity for the 1220 Hz excitation (the standing wave is neglected at this excitation frequency) is shown in Figs. 16(b,d). The higher levels of background noise in the measured velocity field are appreciated, which limits the accuracy of the model.

4.2.2. Phase lag between the hydrodynamic and acoustic waves

One of the main results of Aurégan and Singh [13] is that the oscillations of the transmission coefficient are caused by the interference of the hydrodynamic and acoustic waves at the liner end. It is observed in Fig. 16(c) that the phase of the hydrodynamic and acoustic waves are equal at \( x \approx 71 \text{ mm} \) for the 1500 Hz excitation, which corresponds to a peak of the transmission coefficient (Fig. 2). For the 1220 Hz excitation, corresponding to a minimum of the transmission coefficient, the hydrodynamic and acoustic phases differ by about \( \pi \) rad at \( x \approx 71 \text{ mm} \) (Fig. 16(d)), i.e. they are in counterphase. This supports the argument of Aurégan and Singh and indicates that in fact the interaction obeys a wave interference mechanism, quantified by the relative phase of the two waves in the vicinity of the liner end. It is also appreciated that the instability and acoustic waves are in phase at \( x = 0 \text{ mm} \), as observed above from the phase lags.

Fig. 19. Streamwise (a,b,c) and wall-normal (d,e,f) velocity time signals measured at \( x = 75 \text{ mm} \) (a), at \( y = 4.5 \text{ mm} \) (a,d), \( y = 3.3 \text{ mm} \) (b,e) and \( y = 2.1 \text{ mm} \) (c,f), together with the fundamental harmonic (—) and the sum of the fundamental and second harmonic (— ...) of the Fourier series.

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4.2.3. Detachment of the hydrodynamic wave

The mean flow results have shown that the boundary layer thickens along the porous wall, and the vortex sheet core shifts away from the wall in the downstream liner section. This feature of the mean flow is also observed in the phase-averaged field. The streamwise phase-averaged velocity amplitude profiles upstream ($x = 60\text{ mm}$) and within ($x = 75\text{ mm}$) the region of separation are shown in Fig. 17(a). The analogous plots for the phase are shown in Fig. 17(b). As opposed to the velocity profile upstream of separation, in the separated region the streamwise velocity amplitude has a sharp minimum at $y = 3.3\text{ mm}$, with amplitude approaching zero, and increases again towards the wall. The sharp minimum of the streamwise velocity amplitude carries a change in phase of $\pi$ rad, which is not observed for the wall-normal velocity. This indicates a separation of the instability wave simultaneously with the separation of the mean vorticity core. The high values of streamwise velocity amplitude at $y = 2.1\text{ mm}$, associated to the counter flow (Figs. A1(b,d,f,h,j)), belong to the instability wave itself at the other side of the separated vorticity core. On the other hand, the wall-normal velocity behaves equally in both regions, and it does not suffer a phase change through the vorticity core. It appears to be symmetric with respect to the vorticity core while the streamwise velocity is antisymmetric. The pattern is similar to the case of a free vortex sheet instability.

The relevance of the detachment of the instability wave to the acoustic transmission is not clear. As discussed, it is the interaction between the hydrodynamic and acoustic wave fronts at the liner end which dictate the oscillations in the acoustic transmission as a function of frequency. The hydrodynamic wave is indeed affected by the detachment, and therefore the latter might have an impact on the acoustic transmission.

In the vorticity core the fundamental Fourier harmonic is observed to match poorly the streamwise phase-averaged velocity, and a much better fit results when adding the second harmonic. Figs. 18(a,b) show the streamwise velocity amplitude contours of the fundamental and the second Fourier harmonics, respectively. In the vorticity core the contribution from the second Fourier harmonic is larger than the fundamental. Fig. 19 shows the velocity signals in a lined section where the vorticity core is separated, at three different distances from the wall. Above the vorticity core the second harmonic is...
weak (Figs. 19(a,d)). Within the vorticity core the contribution of the second harmonic to $\nu'$ is larger than the fundamental (Fig. 19(b)). Finally, in the counter flow region the fundamental harmonic becomes larger again (Fig. 19(c)). It can be appreciated how in this region the phase of the fundamental mode has changed by $\pi$ rad with respect to the phase above the vorticity core. The wall-normal velocity is dominated by the fundamental harmonic everywhere.

To explain these results one needs to consider that the separated vorticity core actually ondulates as the instability wave convects downstream. The effect of the ondulation in the local velocity field is visualised in Fig. 20. The streamwise velocity signals along an excitation period inside the ondulation region ($P_{\text{int}}$), above the ondulation region ($P_{\text{ext1}}$), and below the ondulation region ($P_{\text{ext2}}$) are plotted. The shown velocity signals are the ones expected in an infinitely thin free vortex sheet which is linearly unstable [19]. At $P_{\text{int}}$ the velocity signal is maximum at the points of zero vortex sheet displacement (A, B), and is minimum at the points of maximum displacement (C, D, E). This results in a velocity signal at twice the excitation frequency. The phase shift of $\pi$ rad between the velocity at $P_{\text{ext1}}$ and $P_{\text{ext2}}$ can also be appreciated (velocity signals are in counter-phase). The features of this simplified case agree with the measured trends of Figs. 19(a,b,c). This again highlights the similarity between the present case and the case of an unstable free shear layer.

4.3. Turbulent velocity field

The profiles of the rms velocity field measured at $x = 70$ mm are shown in Fig. 21. The curves corresponding to the turbulent fields for the excited and unexcited case are close to each other for both the streamwise and wall-normal components, and their levels are significantly higher than the ones corresponding to the phase-averaged field. The turbulent fluctuations are therefore not significantly affected by the incoming sound and the instability wave. Contradictory to the phase-averaged velocity field, the turbulent velocity fluctuations are higher in the streamwise direction than in wall-normal direction, which is expected from a turbulent boundary layer.

5. Conclusions

This investigation has led to new results concerning the hydrodynamic instability mode over a porous liner in a duct with flow. Most features of the phase-averaged velocity field can be described as the sum of three duct wave modes: the hydrodynamic instability wave, a standing wave and an acoustic wave. The instability wave structure reveals important differences to the case of a locally reacting liner.

The results support the picture described by Aurégan and Singh [13] that the oscillations in the acoustical transmission coefficient as a function of the frequency result from the interference of the hydrodynamic and acoustic wave in the liner downstream section. Depending on the relative phase lag between both waves in the vicinity of the liner end they interfere constructively or destructively, giving rise to the observed ripples of the transmission coefficient.

The mean flow shows an acceleration of the streamwise centreline velocity due to the thickening of the boundary layer. There is a positive wall-normal velocity region in the vicinity of the liner downstream edge, and there the mean vorticity core is shifted away from the porous wall. In parallel, the instability wave also shifts away, inducing a counter flow between the core and the porous wall. Within the vorticity core the second Fourier harmonic is dominant over the fundamental because of its ondulation following the instability wave, similar to the case of the Kelvin–Helmholtz instability of a vortex sheet.

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Appendix A. Phase-averaged velocity fields

The streamwise and wall-normal phase-averaged velocity colourmaps at 5 phases distributed along a period for the 1500 Hz excitation are shown in Fig. A1. The streamwise and wall-normal phase-averaged velocity colourmaps at 5 phases distributed along a period for the 1220 Hz excitation are shown in Fig. A2. Streamwise and wall-normal fundamental Fourier mode colourmaps for the 1500 Hz excitation are shown in Fig. A3. Streamwise and wall-normal fundamental Fourier mode colourmaps for the 1220 Hz excitation are shown in Fig. A4.

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Fig. A1. Colourmaps of $u'(a, b, c, d, e)$ and $v'(f, g, h, i, j)$ (in m/s) at phases $\phi = 1$ ($a, f$), 3 ($b, g$), 5 ($c, h$), 7 ($d, i$) and 9 ($e, j$) for the 1500 Hz excitation [14]. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. A2. Colourmaps of $u'(a, b, c, d, e)$ and $v'(f, g, h, i, j)$ in m/s at phases $\phi = 1$ ($a, f$), 3 ($b, g$), 5 ($c, h$), 7 ($d, i$) and 9 ($e, j$) for the 1220 Hz excitation. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
Fig. A3. Colourmaps of $U$ (a), $\phi_u$ (b), $V$ (c) and $\phi_v$ (d), corresponding to the excitation of 1500 Hz (velocity in m/s and phase in radians). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. A4. Colourmaps of $U$ (a), $\phi_u$ (b), $V$ (c) and $\phi_v$ (d), corresponding to the excitation of 1220 Hz (velocity in m/s and phase in radians). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Appendix B. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jsv.2016.09.034.

References


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