AN ACOUSTIC CRITERION FOR THE WHISTLING OF ORIFICES IN PIPES

P. Moussou*
LaMSID, UMR CNRS
EDF 2832, France

Ph. Testud
LaMSID, UMR CNRS
EDF 2832, France

Y. Auregan
Laboratoire d’Acoustique
de l’Université du Maine,
France

A. Hirschberg
Fluid Dynamics Lab.,
Technische Universiteit
Eindhoven, The Netherlands

ABSTRACT

Whistling due to vortex shedding with lock-in has been extensively studied in the case of cylinders in cross-flows, of flow separation above cavities and of shear layers with flow impingement feedback. Less attention has been given to pressure drop devices in piping systems, which are known to generate high noise levels due to single tones in gas systems, and even in water systems.

Based on recent works of Auregan et Starobinski (1999), an experimental criterion is proposed to evaluate the whistling ability of a pressure drop device in the presence of plane waves acoustic feedback. The idea of the criterion can be summarized as follows: if for a given combination of incident pressure waves, the amount of acoustic power scattered is higher than the incident one, the pressure drop device behaves as an acoustic amplifier, so that whistling can occur if the adequate acoustic boundary conditions are met. The main interest of this criterion is that it depends only on the acoustic scattering matrix of the device, and not on the arrangement of the surrounding pipe.

Results obtained in an air test rig with an inner diameter of 3 cm, a Mach number varying from 10^{-3} to 10^{-1} and a Reynolds number varying from 10^{3} to 10^{5} are reported for single hole orifices. Basing the Strouhal number on the thickness of the orifice and on the average velocity through the hole, thin single hole orifices with sharp angles appear to whistle in a range of Strouhal numbers close to .2. Furthermore, it is shown that a thin orifice with a downstream bevel is prone to whistling, whereas the same orifice with the bevel upstream cannot whistle.

INTRODUCTION

Vortex shedding with lock-in has been extensively studied in the past decades for cylinders submitted to cross-flow, but the whistling of orifices in pipes due to acoustic feedback has been less studied except in overviews [1, 2]. The case of an orifice with a pipe upstream and an open air termination downstream has been studied by Anderson [3, 4], and some industrial studies involving cavitation have been reported [5, 6, 7, 8, 9].

A better understanding of the whistling of orifices in the presence of acoustic feedback can be obtained experimentally by testing an orifice in anechoic conditions, and evaluating its response to pressure waves. Recently, an instability criterion was proposed by Auregan and Starobinski [10], involving acoustic waves of low or high order, and taking into account the influence of the flow velocity on propagation [11]. The purpose of the present study is to apply this criterion to orifices at low Mach numbers, in the framework of plane acoustic waves.

EXPERIMENTAL PROCEDURE

Considered as a ‘black box’ inside an acoustic system, an orifice subjected to incident pressure waves can be described by its plane wave scattering matrix [12], according to:

\[
\begin{pmatrix}
    p_{\text{down}}^+ \\
    p_{\text{up}}^-
\end{pmatrix} = \begin{pmatrix}
    T^- & R^- \\
    R^+ & T^+
\end{pmatrix} \begin{pmatrix}
    p_{\text{up}}^+ \\
    p_{\text{down}}^-
\end{pmatrix},
\]

(1)

the complex coefficients $T^\pm$ and $R^\pm$ being functions of the frequency and of the flow velocity, and the notations being indicated in Fig. 1. The present study requires the determination of these coefficients in order to express a whistling criterion.

![Figure 1: sketch of the incident and scattered acoustic pressure waves](image-url)
Test rig description

The test rig of the Laboratoire d’Acoustique de l’Université du Maine (LAUM) is designed to determine the scattering matrix of passive acoustic devices. The test section consists out of a straight pipe with an inner diameter equal to 30 mm and a total length of 6 m. A constant air flow is generated by a compressor Aerzen Delta blower GM10S (#1 in Fig. 2). The flow rate is measured with a flow meter ITT Barton 7402 (#2 in Fig. 2) with a measurement range from 0.03 to 0.157 m³.s⁻¹. Quasi-anechoic terminations are arranged upstream and downstream (#3 and #9 in Fig. 2) of the measurement area; these terminations are made of a perforated tube and covered with textile. Two loudspeakers are located upstream (#4) and downstream (#8) of the orifice. They generate an acoustical pressure up to 160 SPL in the frequency range 400-4000 Hz.

Acoustic pressures are measured upstream and downstream of the orifice (located at point 6 in Fig. 2) by a couple of 4 microphones B&K 4938 ¼” with Nexus 2690 amplifiers, located respectively at points 5 and 7 in Fig. 2. The distances between sensors are respectively 63.5 mm, 211.5 mm and 700 mm in order to optimize the identification of propagating pressure waves.

The duct is 2 m long upstream of the first series of microphones, so that the flow is fully developed. The temperature is measured with 2 sensors on both sides of the orifice, so that the speed of sound can be estimated. Its value was equal to about 345 m/s in all experiments.

Tested orifices

The tested orifices exhibit a centered single hole with neat sharp angle edges. Their thicknesses \(t\) and hole diameters \(d\) are given in Fig. 3. For the sake of illustration, one orifice with a bevel was tested as well.

Scattering matrix determination

The scattering matrix coefficients are automatically determined with the help of a computer system, details can be found in [13]. During the experiments, the loudspeaker sound level was kept sufficiently low to ensure that the results were not depending on it.

The sound generated by the loudspeakers of the test rig can be fairly described by harmonic pressure waves proportional to \(\exp(i\omega t \pm kx)\). It is worth mentioning the fact that in practical applications, the incident pressure waves are generated by external broadband noise sources of stochastic nature [14, 15], and the formalism of Power Spectrum Density should be used instead. This is yet of no practical consequence, because the replacement of for instance \(p_{up}^+\) by the cross-spectrum of \(p_{up}^+\) with some reference pressure would leave the scattering matrix unaltered. For the sake of simplicity, the harmonic formalism is hence used in the frame of the present study.

Dimensionless representation of the results

The choice was made to plot the results using \(ft/U_d\) as a dimensionless frequency, where \(t\) is the thickness of the orifice and \(U_d\) is the flow velocity in the hole. The other dimensionless numbers of the experiments are the Mach number \(U/c\), expressed in the framework of the study as a function of the pipe velocity \(U\), the ratio of hole diameter to pipe diameter \(d/D\), and the Reynolds number \(UD/\nu\), with self-evident notations.

LOW MACH NUMBER DESCRIPTION OF ORIFICES

Reduction of the scattering matrix to a unique coefficient

Strictly speaking, the four coefficients \(T^e\) and \(R^e\) are independent and need be determined separately. However, as the size of the orifice is small compared to the wavelength, and as the Mach number is low, one expects the acoustic velocity \((p^+ - p^-)/\rho c\) to have the same value upstream and downstream, which brings out:

\[
T^e = 1 - R^e \quad \text{and} \quad T = 1 - R
\]

(2)

A more elaborate condition was tested by making equal the upstream and downstream mass flows. As this procedure introduces correction terms proportionnal to the Mach number which did not alter significantly the results of the present study, the simpler equation (2) is used instead.
A second condition upon the coefficients of the scattering matrix can be obtained by demanding a couple of incident pressure waves with equal signs and amplitudes not to interact with the flow inside the orifice, because a uniform variation of the pressure has no effect if the flow is incompressible:

\[
\begin{pmatrix}
1 \\
1
\end{pmatrix} = 
\begin{pmatrix}
T^+ & R^- \\
R^+ & T^-
\end{pmatrix}
\begin{pmatrix}
1 \\
1
\end{pmatrix}.
\]  
\tag{3}

Solving equations (2) and (3) brings out a scattering matrix with the following expression

\[
\begin{pmatrix}
p_{down}^+ \\
p_{down}^-
\end{pmatrix} = 
\begin{pmatrix}
T & 1-T \\
1-T & T
\end{pmatrix}
\begin{pmatrix}
p_{up}^+ \\
p_{up}^-
\end{pmatrix},
\]  
\tag{4}

where the complex coefficient \( T \) alone describes the passive behavior of an orifice. Such a description reasonably holds in the low Mach number range, as illustrated in Fig. 4. Discrepancies appear for higher Mach numbers, as shown in Fig. 5. Assuming these discrepancies to be of little consequence to the whistling criterion, use is made in the framework of the present study of a unique coefficient \( T \), obtained by averaging the estimations from the four coefficients of the scattering matrix.

As can be seen in Fig. 4 and 5, the Mach number \( U/c \) has an influence upon the transmission coefficient \( T \). Dimensional analysis can be used to determine explicitly this influence. Let the pressure drop law of the orifice be written

\[
\Delta p = \frac{1}{2} \beta U^2 p u_{acou}^2,
\]  
\tag{5}

where \( \Delta p \) is the difference of the steady pressures upstream and downstream, \( \beta \) is a dimensionless coefficient function of \( t/D \) and \( d/D \) [16, 17] and \( p \) is the fluid density. In harmonic regime, the acoustic pressures can be related to the acoustic velocity \( u_{acou} \) by an equation of the same type

\[
p_{up}^+ - p_{down} = B U p u_{acou},
\]  
\tag{6}

where the acoustic velocity is assumed to have the same value upstream and downstream, where \( B \) is a non dimensional function depending on the frequency, and where \( \rho \) and \( U \) are needed by dimension analysis. In quasi-steady regime [18], Eq. (6) can be obtained by linearizing Eq. (5), which indicates that \( B \) is equal to \( \beta \) for low frequencies. In the low Mach number range, one expects \( B \) not to depend on the fluid compressibility, and not to depend on the Reynolds number either. Replacing \( p \) by \( p^+ + p^- \) and \( u_{acou} \) by \( (p^+ - p^-)/pc \) in (6), one gets

\[
p_{down}^+ - p_{up} = \frac{2}{2 + B U/c} (p_{up}^+ - p_{down}^-).
\]  
\tag{7}

Identifying (7) with the second equation of (9), and expressing \( B \) as a function of a dimensionless frequency, of \( t/D \) and of \( d/D \), one gets

\[
T = \frac{1}{1 + \frac{U}{c} B \left[ \frac{t}{U_d}, \frac{t}{D}, \frac{d}{D} \right]},
\]  
\tag{8}

a non-trivial result which makes possible the collapse of experimental data at different flow regimes. This scaling law works reasonably well, as illustrated in Fig. 6 where estimations of \( B \) at different Mach numbers are gathered.
The eigenvalues of (4) can easily be found and one gets

\begin{align}
    p_{\text{down}}^+ + p_{\text{up}}^- &= p_{\text{up}}^+ + p_{\text{down}}^- \\
    p_{\text{down}}^+ - p_{\text{up}}^- &= (2T-1)(p_{\text{up}}^+ - p_{\text{down}}^-). 
\end{align}

(9)

The first equation stands for the equality of the acoustic velocity upstream and downstream, whereas the second one shows that a couple of incident pressure waves with opposite values is amplified by a factor equal to $2T-1$. The whistling criterion can now be formulated. Considering the orifice as an amplifier and the acoustic response of the surrounding pipe as a feedback, instability occurs if the ‘open-loop’ gain is higher than unity, and if some phase condition is met. As the acoustic response of the surrounding system is lower than unity, a necessary condition for whistling to occur is that in a given frequency range, $2T-1$ has a modulus higher than unity. An illustration of the criterion is given in Fig. 7 for an orifice prone to whistling at Strouhal numbers varying from 0.2 to 0.3.

Let $T$ be expressed as a function of $B$ using Eq. (8). The criterion $|2T-1| > 1$ can be rewritten as

$$\left| \frac{U}{c} - B \right| > \left| \frac{U}{c} + B \right|,$$

a condition which is fulfilled when $UB/c$ is closer to $-1$ than to $1$. Such a whistling condition can be expressed under a simpler form as

$$\text{Real} \left( \frac{U}{c} B \right) < 0,$$

(10)

The physical meaning of this criterion can be highlighted considering Eq. 6; in the frequency range where $B$ is a real positive number, the acoustic pressure drop is in phase with the acoustic velocity, and Eq. 6 describes a purely dissipative process. If $B$ were a real negative number instead, acoustic energy would be generated by the orifice. The criterion (10) is then related to the amount of acoustic energy generated or dissipated by the orifice [19], and it is similar to the notion of negative damping in linear instability analysis. The major interest of the criterion (10) is that it can be expressed upon $B$ only, as the Mach number is a positive number. An orifice complying with $\text{Real}(B) < 0$ may theoretically whistle at any Mach number, but the higher the Mach number, the higher the term $|2T-1|$, and the higher the probability of whistling.

As an illustration of the equivalence of the criteria upon $|2T-1|$ and upon $\text{Real}(B)$, the Figures 6 and 7 can be compared: whistling may occur in the range of Strouhal numbers ranging from 0.2 to 0.35 because $|2T-1|$ is higher than unity (Fig. 6) and the argument of $B$ is higher than $\pi/2$. 
RESULTS FOR SHARP EDGE ORIFICES

All orifices exhibit a maximum of the argument of $B$ in the range of Strouhal numbers close to .2. Depending on the $d/D$ and the $t/d$ ratios, this maximum may or may not be higher than $\pi/2$, so that whistling is not possible in all cases. Sometimes, a second maximum is observed for Strouhal numbers close to .6. Detailed results are reproduced in the following figures. Some discrepancies are observed as regards the collapse of the $B$ curves at different Mach numbers, which are probably due to the uncertainty of the velocity measurement at low flow rates.

Low $d/D$ case

The first series of measurements deals with a $d/D$ ratio equal to 0.33. The whistling ability appears to be maximum for $t/d$ close to 0.5, with a Strouhal number equal to 0.25. A second maximum appears for a thicker orifice, at a Strouhal number equal to 1.1, whereas the first maximum decreases below $\pi/2$, preventing whistling at a Strouhal number close to 0.2.

Medium $d/D$ case

For a $d/D$ ratio equal to .66, whistling is again possible for thin orifices at a Strouhal number in the 0.2-0.3 range. The whistling ability vanishes at this Strouhal number for a thicker orifice, and it appears at a Strouhal number close to 0.7.
High d/D case
For larger holes, the argument of $B$ becomes close to $\pi/2$ for all Strouhal numbers. Whistling is not likely to appear except in very reverberating conditions.

RESULTS FOR A BEVEL-SHAPED ORIFICE
Experience shows that vortex shedding with lock-in is dependent on the location where the flow separates [20]. Bevels are hence likely to enhance or to prevent whistling. An attempt to reproduce this effect was made using a bevel-shaped orifice. The bevel has an angle equal to 45°, its thickness is $t/D = 0.17$, the hole diameters are $d/D = 0.33$ and $d_{max}/D = 0.4$, as shown in Fig. 18.

The presence of the bevel upstream totally removes the whistling ability of the orifice as shown in Fig. 19. Whistling is still possible with a bevel downstream, as shown in Fig. 20, though the whistling ability seems slightly lower than in the case without bevel, as shown in Fig. 9 where the thickness and the hole diameter are the same as in Fig. 20.
VALIDATION OF THE CRITERION

In order to validate the criterion, the orifice of Figure 9 was tested in reverberating conditions at different flow rates. An expansion chamber was arranged 137 mm upstream of the orifice, and an unflanged open pipe termination was arranged 270 mm downstream of the orifice, the corresponding boundary conditions being illustrated by the reflection coefficients in Fig. 21.

Due to the length of the pipe from one end to the other, one expects the first natural frequency of the system to be close to some 170 Hz, and the higher natural frequencies to be multiples of this frequency. Hence, in the range 1000-3000 Hz and for a Mach number varying from 0.0046 to 0.023, there should always exist an acoustic natural frequency such that the Strouhal number is in the range 0.2-0.4, i.e., in the range where whistling is likely to occur (see Fig. 9).

Whistling actually occurs in all cases, as shown in Fig. 22 to 25 where the pressure PSD downstream of the orifice is shown for different Mach numbers. The first natural frequency of the pipe is 152 Hz, and whistling occurs at 1000 Hz, 1800 Hz, 2200 Hz and 2900 Hz.
At intermediate values of the Mach number, the whistling occurs alternatively at several frequencies, as illustrated in Fig. 26 and Fig. 27.

In order to summarize the results, the dimensionless r.m.s. values of the pressure peak are plotted as a function of the Strouhal number in Fig. 28 for all flow conditions. It clearly appears that whistling occurs at a Strouhal number in the range 0.2-0.4, consistent with the whistling criterion in Fig. 9. It should be noted that the increase in the 2300 Hz peak amplitude at a Strouhal number of .5 is actually due to an harmonic of the 1000 Hz peak.
CONCLUSION

An acoustic criterion for evaluating the whistling ability of an orifice was proposed. It is found that sharp edges orifices with a thickness to hole diameter ratio in the range 0.2-0.5 are prone to whistling, for Strouhal numbers close to 0.2.

The criterion is validated by tests: in reverberating conditions, an orifice generates single tone noise in the Strouhal number range corresponding to the criterion.

The main advantage of the criterion is that it does not depend on the acoustic boundary conditions of the test rig; the determination of the scattering matrix in non-whistling conditions makes the whistling ability of the orifice in an other acoustic surrounding predictable. It can hence be used to design whistle-free orifices.

Further work is needed to test the criterion at other Reynolds numbers, and with other fluids so as to validate the independence of the criterion with the Mach number. Another consists in assembling several orifices to determine the conditions where the whistling ability is enhanced.

REFERENCES


