Measurement of the nonlinear behavior of acoustical rigid porous materials

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The measurement of the flow resistivity of porous materials shows that two types of behavior can be observed, depending on the value of a Reynolds number based on the porous material microgeometry. For Reynolds numbers (Re) smaller than a critical Reynolds number, the increase of the resistivity is quadratic in Re. For Re larger than this value the increase is linear in Re (Forchheimer’s law). A comparison between acoustic measurements and an equivalent fluid model shows that the main effect of high sound level on sound propagation through rigid porous materials is the variation of the flow resistivity. © 1999 American Institute of Physics. [S1070-6631(99)00106-3]

I. INTRODUCTION

In a number of practical applications, porous materials that are used to attenuate sound are exposed to intense sound pressure levels (SPL). In this case, nonlinear behaviors appear in the sound propagation through the media. This can be due either to nonlinearities in the flow field in the case of rigidly framed material\textsuperscript{1} or to nonlinearities in the stiffness of the frame.\textsuperscript{2}

In this paper, only rigidly framed porous materials will be considered. For this kind of media, the linear sound propagation can be described by mean of an equivalent fluid with an effective density and an effective compressibility which are complex values depending on the frequency (see, e.g., Johnson et al.\textsuperscript{3} and Lafarge et al.\textsuperscript{4}). The effective characteristics of the material can be obtained with the help of six parameters: the porosity $\Phi$, the tortuosity $\alpha_\Phi$, the viscous and thermal permeability $k_0$ and $k_0'$, the viscous and thermal characteristic lengths $\Lambda$ and $\Lambda'$. In this paper, the static thermal permeability is approximated\textsuperscript{4} by $k_0' = \Phi \Lambda'^2/\delta$ and the static viscous permeability $k_0$ is only used in the definition of the resistivity $\sigma = \mu / k_0$, where $\mu$ is the viscosity of the fluid. A previous paper on nonlinearity in rigid porous media\textsuperscript{5} leads to the conclusion that only resistivity is involved in the nonlinear effects.

New experimental results concerning the nonlinear deviation of the resistivity are given in Sec. II. Particular precautions are taken to obtain accurate results at very low flow rates in the porous media. At low flow rates, the problem is amenable to asymptotic analysis\textsuperscript{5,6} and a good match between the experimental results and the results of the calculations and numerical simulations is observed. For higher flow rates, the Forchheimer behavior,\textsuperscript{5,6} already observed by other authors, is observed experimentally. The measurements reported in Sec. II are used to predict the nonlinear acoustical behavior of porous materials described in Sec. III. The predictions of the equivalent fluid model are compared to new experimental results obtained by acoustic measurements at high amplitudes. The agreement between predictions and measurements shows that the variation of the flow resistivity is the main effect of high sound level on sound propagation through rigid porous media.

II. RESISTIVITY MEASUREMENTS

The resistivity of a porous media is defined by $\sigma = \Delta p / V L$ where $\Delta p$ is the pressure drop induced by a porous material sample of thickness $L$ when a continuous flow of velocity $V$ is passed through it (the seepage velocity $V$ is equal to the flow rate $Q$ divided by the section of the material sample $S$).

Darcy’s law states that, for small flow rates, the pressure drop is proportional to the seepage velocity and to the length of the sample. Thus, in Darcy’s approximation, the resistivity is constant: $\Delta p = \sigma_0 V L$.

Some deviations to Darcy’s law appear when the seepage velocity increases. The relevant parameter to describe these variations of resistivity is the Reynolds number defined here by $Re = 2 V_p / \nu$ where $\Lambda$ is a viscous characteristic length which depends on the microgeometry of the pores,\textsuperscript{3} $V_p = (Q / S) / \Phi$ is the mean flow velocity in the pores and $\nu$ is the kinematic viscosity.

For high Reynolds number (Re larger than unity), it is commonly accepted that the resistivity increases linearly with the Reynolds number

$$\sigma = \sigma_0 (1 - \delta) + C_1 Re.$$  \hspace{1cm} (1)

This empirical behavior is classically referred to as the Forchheimer’s law.

In spite of its practical interest in filtration and in acoustics, the low Reynolds number zone has not often been studied experimentally. It has now been theoretically demonstrated by asymptotic analysis that, for typical microgeometry used in acoustical porous materials, the deviation from Darcy’s law is quadratic in Reynolds number.
when the Reynolds number is much smaller than unity (see, e.g., Firdaouss et al. and Rojas and Koplik for a careful treatment of the problem and a review of the literature).

The resistivity measurements of two porous materials used in acoustical applications are presented below. The materials were chosen to have very different pore microgeometries. One is a plastic foam and the other is an agglomerated rubber made with pieces of about 1 mm in size. The acoustical parameters of those materials and the length of the used samples are indicated in Table I.

In order to measure the resistivity, a sample is inserted in a tube (diameter 44.5 mm). The pressure drop is measured on both sides of the sample by differential pressure transducers. Three pressure transducers (EFFA GA 75, 0–0.1 mb, 0–1 mb, 0–20 mb) are used to cover the whole range of measured pressures with precision. The flow rate is deduced from the pressure drop through a calibrated resistance inserted in the same tube. The flow rate is accurately regulated and stabilized by a tank of about 0.5 m$^3$ and the setup is located in a room without external perturbations (quiescent atmosphere, no noise,...) to obtain precise results at very low flow rates in the porous media.

The variations of the resistivity are depicted in Figs. 1 and 2 as a function of the Reynolds number. For high Reynolds numbers ($Re > 1$), the variation of the resistivity is linear according to Forchheimer’s law. For low Reynolds numbers a deviation from this law can be observed. This behavior is emphasized in the enlargements in Figs. 1 and 2. It can be deduced from those experimental data that the resistivity increases as the square of the Reynolds number up to a critical Reynolds number $Re_c$. The dimensionless variables $Re_c$, $C_2$, $\delta$, and $C_1$ used in Eqs. (1) and (2) are given in Table II. Those coefficients lead to a slope continuity of the fitted curves when $Re_c = Re_c$ (Fig. 3).

The quadratic coefficient $C_2$ is about 10 times greater for the agglomerated rubber than for plastic foam and the critical Reynolds number $Re_c$ is 6 times smaller for the agglomerated rubber. This points to a larger sensitivity of the agglomerated rubber to nonlinear effects, which was expected because of the shape of the microgeometry. In an agglomerated rubber, the pores are made of cavities connected together by small channels. Thus, the velocity gradients are much larger than in the plastic foam. Likewise the ratio between the nonlinear term and the viscous term in the Navier–Stokes equation is expected to be larger for the agglomerated rubber.

### III. Acoustic Measurements at High SPL

A material sample is inserted in a tube in which linear acoustical plane waves are propagated (see Fig. 3). On one side (side 1), a loudspeaker provides an acoustic level up to 150 dB (Ref. 20 $\mu$Pa) in the frequency range 50–800 Hz. The acoustic pressure in tube 1 is written $p_1 = p_1^+ e^{-jkx} + p_1^- e^{jkx}$ where $p_1^+$ and $p_1^-$ are the incident and reflected pressures.

### Table I. Acoustical parameters ($\Phi$, tortuosity $\alpha$, viscous length $\Lambda$, thermal length $\Lambda'$, linear resistivity $\sigma_0$, and sample thickness $L$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\Phi$</th>
<th>$\alpha$</th>
<th>$\Lambda$</th>
<th>$\Lambda'$</th>
<th>$\sigma_0$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic foam</td>
<td>0.98</td>
<td>1.3</td>
<td>0.9x10^-4</td>
<td>2.4x10^-4</td>
<td>6004</td>
<td>43.7x10^-3</td>
</tr>
<tr>
<td>Agglomerated rubber</td>
<td>0.47</td>
<td>1.8</td>
<td>1.1x10^-4</td>
<td>7.5x10^-4</td>
<td>7013</td>
<td>53.0x10^-3</td>
</tr>
</tbody>
</table>

*Units are m.

*Units are kg m$^{-2}$ s$^{-1}$.

\[ \sigma = \sigma_0(1 + C_2 Re^2) \quad (2) \]
pressures on side 1 of the sample, \( j = \sqrt{-1} \), \( k \) is the wave number (taking into account viscothermal attenuation), and \( x \) is the axial distance from side 1 of the sample. Three microphones in tube 1 allow an overdetermined estimation of \( p_1^+ \) and \( p_1^- \). The overdetermination is used, with a least square method, to increase the accuracy of the experimental results. The acoustic velocity \( u_1 \) on side 1 of the sample is computed by \( u_1 = (p_1^+ - p_1^-) / Z_e \) where \( Z_e \) is the characteristic impedance of air. On the other side (side 2), three other microphones are used so as to measure the transmitted pressure \( p_2^+ \) and the pressure reflected from the tube termination \( p_2^- \).

To avoid standing waves in the tubes, a weakly reflective termination is used on side 2 (the weak reflectivity of this termination is taken into account in the model). As a result, the acoustical amplitude is roughly constant in the tube and the acoustic pressure and the velocity never vanishes at the sample location, throughout the frequency range of interest.

The reciprocity and the symmetry of the measured element imply that the transmission \( T \) and reflection \( R \) coefficients satisfy \( p_2^+ = T p_1^+ + R p_2^+ \) and \( p_1^+ = T p_2^+ + R p_1^- \). Thus, the coefficients \( R = (p_1^+ p_1^- - p_2^+ p_2^-) / (p_1^+ p_2^- - p_2^+ p_2^-) \) and \( T = (p_1^+ p_2^- - p_1^- p_2^+) / (p_1^+ p_2^- - p_2^+ p_2^-) \) can be computed from the microphone data as a function of the frequency and of the amplitude of the acoustic velocity \( |u_1| \). The transmission coefficient represents the ratio of transmitted to incident pressures: \( T = p_2^+ / p_1^+ \) when the tube 2 is anechoic \( (p_2^- = 0) \).

The equivalent fluid model is used to describe the acoustical behavior of the materials. Five material parameters: \( \sigma \), \( \Phi \), \( \alpha_e \), \( \Lambda \), and \( \Lambda' \) are needed to calculate an effective density \( \rho_e \), an effective compressibility \( \beta_e \) depending on the frequency \( f \). From those values, the effective wave number \( k_e = 2 \pi f (\rho_e \beta_e)^{1/2} \) and the effective characteristic impedance \( Z_e = \rho_e / \beta_e \) are deduced. The value of the transmission coefficient is then given by \( T = (\cos(k_e L) + j((k_e^2 + 1) / 2 \xi) \sin(k_e L))^{-1} \) where \( L \) is the thickness of the sample, \( \xi = Z_e / \Phi Z_e \), and \( Z_e \) is the characteristic impedance of air.

The amplitude of measured transmission coefficients for the two materials are depicted as symbols in Figs. 4 and 5 for three different frequencies as a function of the Reynolds number. For the frequencies under investigation and for the sample thickness used, the acoustic velocity is almost constant through the sample: \( |u_1| = |u_2| \) and the acoustic Reynolds number is computed by \( \text{Re} = 2 \pi V_p / \nu \) with \( V_p \) as a function of Reynolds number in the pores \( \text{Re} \). In Figs. 4 and 5, the resistivity used in the model is the fit of the data given in Sec. II. The four remaining parameters given in Table I have been measured using another experimental setup and are assumed to remain unchanged when the level increases.

The agreement between the measurements of \( R \) and \( T \) and the model is good for both materials, for all the Reynolds numbers and for all the frequencies under investigation (see Pachebat for the extended results and the complete setup description). A transition in behavior in porous materials takes place at the frequency for which the acoustic boundary layer is equal to the viscous characteristic length: \( \delta_v = (2 \nu / 2 \pi f)^{1/2} \approx \Lambda \) where \( \nu \) is kinematic viscosity. This transition frequency is \( f_t \approx 590 \text{ Hz} \) for the agglomerated rubber...

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**TABLE II. Nonlinear dimensionless parameters of the materials.**

<table>
<thead>
<tr>
<th>Material</th>
<th>( R_c )</th>
<th>( C_2 )</th>
<th>( \delta )</th>
<th>( C_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic foam</td>
<td>3.2</td>
<td>( 1.47 \times 10^{-2} )</td>
<td>0.132</td>
<td>( 8.85 \times 10^{-2} )</td>
</tr>
<tr>
<td>Agglomerated rubber</td>
<td>0.5</td>
<td>( 13.0 \times 10^{-2} )</td>
<td>0.022</td>
<td>( 11.0 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

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**FIG. 3. Schematic description of the acoustic setup.** (1) loudspeaker, (2) microphones, (3) porous material sample, (4) weakly reflective termination.
ber and $f_t \approx 395$ Hz for plastic foam. The good agreement at very low frequencies ($f < f_t$) was expected since a quasi-static flow model can be applied in the pores. According to the equivalent fluid model, the influence of the resistivity decreases with frequency and the acoustical behavior is then only governed by the high-frequencies parameters: $\alpha_\infty$, $\Lambda$, and $\Lambda'$. If one assumes that only the resistivity depends on the sound level, the influence of the Reynolds number on the acoustical behavior decreases with frequency. This fact is clearly illustrated in Fig. 5. This effect at high frequencies is surprising and will need further theoretical investigations.

IV. CONCLUSIONS

It was demonstrated experimentally that the behavior of a porous media’s resistivity differ significantly over two seepage velocity regions. Below a critical Reynolds number of the order of unity, the resistivity varies as the square of the Reynolds number. Above this critical Reynolds number, the variation is linear, according to the classical Forchheimer law. The critical Reynolds number and the quadratic coefficients of resistivity depend on the micro-geometry of the pores.

The comparison between acoustic measurements and an equivalent fluid model shows that the increase in resistivity describes most of the nonlinear effects appearing in rigidly framed porous media when the sound level increases. Further theoretical investigations are needed to explain the weak dependence of high-frequency parameters to high sound level.

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4 D. Lafarge, P. Lema