Parametric LES/SI Based Aeroacoustic Characterization of Tandem Orifices in Low Mach Number Flows

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Summary
Acoustic scattering and noise generation in tandem orifice configurations are examined and modelled. The modelling is carried out by a combination of Large Eddy Simulation (LES) and System Identification (SI). Hereby, an acoustically excited LES is performed. Afterwards, acoustic data series extracted from the LES domain are post-processed by means of parametric SI methods to concurrently identify both acoustic scattering and noise sources. Adopting a system theory perspective, the scattering matrix is represented by the so-called “plant model” whereas the noise sources are described using a “noise model”. Here, two tandem orifice configurations are investigated to assess the influence of the distance between the two singularities on the acoustic power produced by the whole system. The acoustic power generated or dissipated across the double orifices is evaluated by means of the so-called whistling criterion. Hereby, the whistling potentiality of the two tandem orifice configurations is computed from the identified scattering matrix. The deduced acoustic power is subsequently compared against the whistling potentiality of one single orifice composing the ducted systems to evince the influence of the distance between the singularities. Numerical results are validated against experimental data measured for the same geometries and for the same flow.

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1. Introduction
The aeroacoustic characterization of duct systems composed by multiple elements represents an intricate problem in current research. Acoustic feedbacks due to interactions among components of a given set-up are often responsible of whistling or intense broadband noise sources. Beside the noise annoyance, in the extreme cases, these acoustic interactions induce acoustic fatigue or structural failure in the overall pipe system. Therefore, the development of methodologies to assess, analyse and systematically model such aero-acoustic phenomena is of crucial importance.

The characterization of complex acoustic systems is often done by means of a divide and conquer strategy. Hereby, the whole system is divided in simpler acoustic subsystems (also called “elements”) interconnected at their respective interfaces. These are usually called “ports” of the acoustic element. The acoustic properties of the original system are therefore retrieved by interconnecting the elements into a so-called “network model” [1, 2]. The complex aeroacoustic characterization is therefore reduced to the characterisation of “simpler” acoustic elements. These are modelled mathematically by adopting the so-called “Multiport method” [2, 3] as
\[ y(\omega) = S_{M}(\omega)u(\omega) + v_{N}(\omega) \]
where $y(\omega)$ are the responses of the acoustic element to given acoustic inputs $u(\omega)$ with frequency $\omega$. On the one hand, the acoustic scattering across a given element is described by means of its scattering matrix $S_{M}(\omega)$. On the other hand, noise sources are represented through a noise vector $v_{N}(\omega)$.

Experimentally, the acoustic scattering matrix is usually assessed by exciting separately the configuration of interest at each port with harmonic signals of frequency $\omega$. The response of the system is extracted by means of the so-called “Multi-microphone method” [3] at the set-up terminations. At first, pressure signals measured at microphone arrays positioned upstream and downstream the duct elements of interest are transformed in the Fourier domain. Subsequently, amplitudes and phases of the characteristic acoustic waves entering and leaving the experimental setup are retrieved by solving a linear minimum mean square error problem [3]. Once the characteristic acoustic waves are known, the coefficients of the scattering matrix can be
retrieved dividing the Fourier transform of the outgoing waves by the Fourier transform of the imposed acoustic signals. The noise sources are assessed in a following step, once the acoustic scattering across the elements and the acoustic reflections at the terminations are known. This is done either by measuring the internal pressure and velocity fluctuations without any imposed external excitation [3] or by imposing a further acoustic state obtained by simultaneously exciting the system at each termination [4].

The numerical modelling of the acoustic scattering may follow the same principles as the experimental approach. The aero-acoustic properties of a given duct system are assessed from the measurements of its inputs and of its outputs. Here, the acoustic field is simulated by solving the Navier-Stokes equations with different levels of approximation. Methods based on Linearized Euler Equations (LEE) [5] and Linearized Navier-Stokes Equations (LNSE) [6, 7] assess the acoustic scattering by harmonically exciting a numerical simulation linearized around a mean flow field. Hereby, a good estimation of the acoustic scattering is achievable. However, since the simulations are carried out w.r.t. a steady flow field, broadband noise sources due to turbulence cannot be assessed from first principles.

The assessment of the acoustic scattering at an element in an unsteady flow field can be performed by means of the so-called LES/SI method developed by Polifke and co-workers, e.g. [8, 9], which combines Large Eddy Simulation (LES) and System Identification (SI) techniques. Hereby, an LES excited with a broadband acoustic signal is performed. The acoustic data series extracted from the LES domain are afterwards post-processed by means of SI to model the acoustic scattering. The use of a broadband signal and SI considerably reduces the computational costs of LES, compared to multiple single-frequency excitations as used in the linearized approach (LEE, LNSE).

The original approach based on correlation analysis and Wiener-Hopf Inversion (WHI) [10] does not model the noise sources. To solve this issue Sovardi et al. [11, 12] proposed a modified version of the original LES/SI method based on a parametric identification and Prediction Error Methods (PEM) [13]. It affords a concurrent characterization and modelling of both acoustic scattering and noise sources. The modelling of the latter is of fundamental importance in the development of 1D-acoustic network models taking into account the presence of internal noise. With this approach, a single LES excited with a broadband signal is required to completely characterize the element. This considerably reduces the computational efforts compared to separately identify acoustic scattering and noise sources as done in the experiments.

In this work the parametric LES/SI approach is used to study the aeroacoustic properties of two tandem orifice configurations. These represent a good benchmark test to analyse non-compact duct systems composed by multiple acoustically interacting elements. Indeed, the acoustic interactions between the two orifices influence the acoustic power produced by the overall system. Here, the phase delays introduced by the duct between the two duct singularities play a significant role. The acoustic interactions between the two singularities are analysed in terms of potential whistling. The numerical predictions obtained by means of the LES/SI are directly compared against experiments carried out on the set-up described in Testud et al. [14]. Hereby, the experimental assessment of the acoustic scattering matrix has been performed by imposing multiple single-frequency excitation signals at the terminations of the test-rig. Noise sources are assessed by performing a full source characterization [3], measuring the acoustic fluctuations at the microphones without any acoustic excitation and removing the contribution of the acoustic reflections at the boundaries.

Theoretically, the identification methods presented in this work can be also extended to experimental analysis (see e.g. [15]). This requires the excitation of the experimental test-rig with acoustic broadband signals and the extraction of the acoustic travelling waves in the time domain. Hereby, loudspeakers must afford acoustic excitations with a good Signal to Noise Ratio (SNR) for all the frequencies of interests to discriminate between intrinsic (broadband) noise and deterministic response of the system. Moreover, the multi-microphone method [2] has to be reformulated to handle time dependent data series [16]. Nevertheless, this formulation is extremely sensitive to the measurement noise at the microphone channels. Significant errors in the estimation of the characteristic acoustic waves may occur for small measurement noise in the channels of the acquisition system. Finally, non-reflecting terminations are required to not have highly correlated input channels, when a SI based on WHI is employed.

Taking into account the aforementioned considerations, the well established experimental procedure based on sinusoidal excitation signals and multi-microphone method in the frequency domain has been preferred for the measurement campaign.

2. Geometry and operating conditions
The acoustic configurations analysed in this work consist of a duct of diameter $D$ inside which two orifices are positioned at distances $L$. A graphic representation is given in Figure 1. The values of the respective geometrical variables are reported in Table I. The two configurations considered are named L41 and L70 in relation to the distances between the two singularities $L = 41 \text{ mm}$ and $L = 70 \text{ mm}$, respectively. Each orifice has a diameter $d$ and a thickness $t$. The study is carried out in the plane wave frequency range. Before and after the double orifice test section one duct upstream of length $l_u$ and one duct downstream of length $l_d$ have been positioned. They have been taken long enough to let the local high-order acoustic modes decay in both experimental and numerical campaigns.

The configurations are analysed for an inlet bulk velocity $U_{\text{bulk}} = 9 \text{ m/s}$. In the LES a reference pressure $P_{\text{ref}} = 101325 \text{ Pa}$ and a reference temperature $T_{\text{ref}} = 298.15 \text{ K}$ have been assumed. In the upstream duct section, these
operating conditions correspond to a Reynolds number $\text{Re} \simeq 18000$ and a Mach number $\text{Ma} = 0.026$.

3. The acoustic element

The Multiport description [2] is briefly reviewed in application to the double orifice configurations in Figure 1. Hereby, the aeroacoustic properties of the cases L41 and L70 are mathematically represented by the respective acoustic scattering matrix and noise vector. From the Multiport representation several further acoustic properties of the element can be deduced. In this work, the acoustic whistling potentiality of tandem orifice configurations is analysed. This has been used to assess the influence of the distance between the two singularities on the whistling potentiality of the configurations.

3.1. Multiport method

The acoustic scattering and the noise sources of a ducted element (see Figure 1), in the linear acoustic regime, at frequencies below the cut-off frequency of the duct, can be expressed mathematically by the so-called scattering matrix and noise vector [2, 3].

$$
\begin{bmatrix}
    f_d(\omega) \\
    g_d(\omega)
\end{bmatrix} = \begin{bmatrix}
    T_{ud}(\omega) & R_{ud}(\omega) \\
    R_{ud}(\omega) & T_{ud}(\omega)
\end{bmatrix} \begin{bmatrix}
    f_s(\omega) \\
    g_s(\omega)
\end{bmatrix}.
$$

where $\omega$ is the angular frequency of interest.

The terms $f_d(\omega)$ and $g_d(\omega)$ indicate two characteristic acoustic waves impinging upon the downstream and upstream termination, respectively. The variables $f_s(\omega)$ and $g_s(\omega)$ represent two characteristic acoustic waves ingoing in the ducted element from the upstream and downstream termination, respectively. The noise sources are represented by the noise vector containing the characteristic acoustic waves $f_s(\omega)$ and $g_s(\omega)$. For a given angular frequency $\omega$, the scattering of acoustic waves across the elements is modelled by means of the transmission and reflection coefficients of the acoustic scattering matrix in Equation (2). Hereby, $T_{ud}(\omega)$ indicates the transmission in the downstream direction of an acoustic wave coming from the upstream termination. $T_{ud}(\omega)$ represents the transmission in the upstream direction of an acoustic wave coming from the downstream termination. In the same way, the terms $R_{ud}(\omega)$ and $R_{ud}(\omega)$ indicate the acoustic reflections upstream and downstream the element, respectively. Hence, for a given angular frequency $\omega$, the terms of the scattering matrix relate the outgoing acoustic waves of the system to the incoming ones. In the linear regime, Equation (2) represents a Linear Time Invariant (LTI) system, whose outputs are $f_d(\omega)$ and $g_d(\omega)$ and whose inputs are $f_s(\omega)$ and $g_s(\omega)$. The outputs of the system are therefore a linear superposition of the acoustic waves scattered at the element and of the noise sources due to turbulence. The latter, in the linear regime, are independent from the system inputs, i.e. from an incoming acoustic fluctuation.

3.2. The whistling criterion

The goal of this work is to investigate the acoustic interactions between two orifices in a duct. Hereby, the increment or the dissipation of the acoustic power across the two tandem elements is investigated. This is assessed by using the so-called whistling criterion developed by Aurégan and Starobinsky [17]. It estimates the whistling potentiality and hence the acoustic power generated by a given configuration from its scattering matrix.

This is done by reformulating the Equation (2) in energy form

$$
\begin{bmatrix}
    \Pi^f_1(\omega) \\
    \Pi^f_2(\omega)
\end{bmatrix} = \begin{bmatrix}
    T_{ud}(\omega) & 1/\text{Ma} \cdot R_{ud}(\omega) \\
    1/\text{Ma} \cdot R_{ud}(\omega) & T_{ud}(\omega)
\end{bmatrix} \begin{bmatrix}
    f_s(\omega) \\
    g_s(\omega)
\end{bmatrix}.
$$

where $\text{Ma}$ is the Mach number and $\Pi_{ud}(\omega)$ represents the mechanical energy associated to the acoustic fluctuations. These are related to the characteristic acoustic waves $f$ and $g$ in Equation (2) as follows

$$
\Pi_{ud}(\omega) = 1 + \text{Ma} \cdot \rho c f_{ud}(\omega)
$$

where $\rho$ is the fluid density and $c$ is the speed of sound.

The whistling potentiality of an element is evaluated by computing the eigenvalues of the matrix

$$
\begin{bmatrix}
    \xi_{\max}(\omega) \\
    \xi_{\min}(\omega)
\end{bmatrix} = \text{eig} \left( I_d - S(\omega)^T S(\omega) \right).
$$

where $I_d$ is the identity matrix, $\xi_{\max}(\omega)$ and $\xi_{\min}(\omega)$ are the minimum and maximum eigenvalue, respectively. In particular, if $\xi_{\min}(\omega) < 0$ the acoustic element generates acoustic power, whereas if $\xi_{\min}(\omega) > 0$ acoustic waves are dissipated across the domain. The lower $\xi_{\min}(\omega)$, the higher the whistling potentiality of the ducted system and consequently the higher the acoustic power generated.
4. System modelling of acoustic elements

A preliminary step to understand the parametric SI approach is to reconsider the acoustic element in a system theory perspective. Indeed, the 2-Port configuration reported in Equation (2) is a $2 \times 2$ Multiple Input Multiple Output (MIMO) system, with two inputs and two outputs. To enhance comprehension of the mathematical issues concerning the system modelling, the discussion is firstly applied to the coefficient $T_{ud}$ of the scattering matrix and to the term $f_d$ of the noise vector. Hereby, Equation (2) is simplified as

\[ f_d(\omega) = T_{ud}(\omega)f_s(\omega) + f_i(\omega) \]  

(7)

where $f_d(\omega)$ is the deterministic response in the frequency domain (scattered wave, transmitted wave) of the system to an incoming acoustic wave $f_s(\omega)$. In the system theory, $f_i(\omega)$ represents the Fourier transform of the stochastic part (noise sources) of the system. In the linear regime, the output of the system $f_d(\omega)$ is obtained by simply summing the deterministic response $f_d^* (\omega)$ of the configuration with the stochastic noise sources $f_i(\omega)$ as shown in Figure 2. Equation (7) represents a Single Input Single Output (SISO) system. This simplification describes the situation in which the 2-Port is excited only at the upstream termination and the response of the system is measured at the downstream duct.

In the next section, the modelling of simplified SISO subsystems presented in [12] is reviewed. Afterwards, the same concepts are applied to model the whole 2-Port in section 4.2.

4.1. Plant and Noise model

Rewriting Equation (7) in the discrete time domain one has

\[ f_d[k] = f_d^*[k] + f_i[k]. \]  

(8)

where $k$ is the time state related to the physical time $t = k\Delta t$, with $\Delta t$ sampling time interval. Thus, at a given state $k$ the output of the system $f_d[k]$ is due to the contribution of both deterministic response $f_d^*[k]$ and noise sources $f_i[k]$. Following [11, 12] Equation (8) can be related to the system input in the discrete time domain $f_d[k]$ as

\[ f_d[k] = G_{ud}(q)f_s[k] + H_j^*(q)e[k]. \]  

(9)

where $e[k]$ is a Gaussian White Noise (GWN) of null mean value and variance $\sigma_e^2$. $G_{ud}(q)$ is the so-called Plant Model. It models the deterministic response (scattered acoustic wave $f_d^*[k]$) of the system to a given input (incoming acoustic wave $f_s[k]$); $H_j^*(q)$ is the so-called Noise Model. It models the stochastic properties of noise sources (turbulent noise) of a system by filtering a GWN $e[k]$. The variable $q$ is known as time-shift operator. It relates two time states $[k]$ and $[k - m]$ of a given input or output as follows

\[ f_a[k - m] = q^{-m}f_a[k]. \]  

(10)

Equation (9) represents therefore the complete response of the SISO subsystem in the discrete time domain by introducing proper filters $G_{ud}(q)$ and $H_j^*(q)$. This is graphically shown in Figure 3.

The mathematical formulation of the noise and of the plant model are not unique. Here, different mathematical functions can be adopted [10, 13]. As it will be shown in section 5.1, in this work a polynomial parametric formulation based on the Box-Jenkins model [10] is used. This has been chosen for its generality and thus for its capacity to independently model both noise and plant model. Alternative mathematical formulations of $G_{ud}(q)$ and $H_j^*(q)$ are described in the literature [10, 12].

The acoustic transmission coefficient $T_{ud}$ can be retrieved by simply transforming the plant model in the frequency domain. Indeed, by taking the discrete Fourier transform of $f_d^*[k]$ and considering Equation (9) one obtains

\[ f_d^*(\omega) = G_{ud}(q^o)p_{ud}(\omega) = T_{ud}(\omega)f_s(\omega). \]  

(11)

In other words, by assuming $q = q^o$ it is possible to retrieve the desired transmission coefficient of the scattering matrix from the plant model of the system.

The noise sources $f_i(\omega)$ can be educed in the same way from the discrete time system in Equation (9) by computing the Fourier transform of $f[k]$ as

\[ f_i(\omega) = H_j^*(q^o)e(\omega), \quad e[k] \sim \mathcal{N}(0, \sigma_e). \]  

(12)

However, in aeroacoustic applications, noise sources are mostly analysed in terms of Power Spectral Density
Scattering matrix: By properly choosing a GWN with a unitary variance $\sigma^2_e = 1$, Equation (14) can be rewritten as

$$\Phi_J^s(\omega) = \left\| H_J^s(e^{j\omega}) \right\|^2 e[k] \sim \mathcal{N}(0, \sigma_e).$$

By properly choosing a GWN with a unitary variance $\sigma^2_e = 1$, Equation (14) can be rewritten as

4.2. From SISO to MIMO

The previous modelling of the SISO subsystem can be easily extended to model acoustic configurations with multiple input and output channels. For a MIMO 2-Port system analysed in this work, this can be written as

$$\begin{bmatrix} f_d[k] \\ g_d[k] \end{bmatrix} = \begin{bmatrix} G_{ud}(\omega) & G_{dd}(\omega) \\ G_{ad}(\omega) & G_{dd}(\omega) \end{bmatrix} \begin{bmatrix} f_d[k] \\ g_d[k] \end{bmatrix} + \begin{bmatrix} H_J^u(\omega) \\ 0 \end{bmatrix} e_1[k] + \begin{bmatrix} 0 \\ H_J^d(\omega) \end{bmatrix} e_2[k].$$

The previous equation corresponds to the block scheme depicted in Figure 4. By transforming in the frequency domain Equation (16) and considering as inputs GWN $e_1[k], e_2[k]$ such that $\sigma_{e1} = \sigma_{e2} = 1$, the following relations hold:

- Scattering matrix:
  $$T_{ad}(\omega) = G_{ad}(e^{j\omega}), \quad R_{ud}(\omega) = G_{ud}(e^{j\omega}),$$
  $$R_{dd}(\omega) = G_{dd}(e^{j\omega}), \quad T_{da}(\omega) = G_{da}(e^{j\omega}).$$

- Noise sources PSD:
  $$\Phi_J^s(\omega) = \left\| H_J^s(e^{j\omega}) \right\|^2.$$  

Equation (16) represents the modelling of an acoustic element through a general system-theory perspective. Hereby, the noise and the plant models can be considered as dynamic filters characterizing the noise sources and the acoustic scattering. As already said for the SISO subsystem in Equation (9) any explicit mathematical formulation of $G_{ad}(q)$ and $H_J^u(q)$ has not been given yet. Here, is where system identification comes into play.

The goal of SI is indeed the definition of proper mathematical structures and algorithms to reproduce the dynamics of a given system of interest from the measurements of its inputs and outputs. In the next section, a parametric mathematical formulation of the noise and plant model is given. Hereby, the filters are mathematically expressed as rational polynomials whose parameters are unknown. These are estimated by exploiting acoustic data series extracted from an excited LES.

5. A parametric LES/SI method

The numerical identification of the acoustic scattering matrix and of the noise sources of tandem orifice configurations is performed by means of a parametric LES/SI method. Firstly, a broadband acoustically excited Large Eddy Simulation is carried out. Afterwards, acoustic data series extracted from the LES are post-processed through parametric System Identification [18, 10, 19, 13] techniques to characterize the acoustic properties of the elements of interest. This requires the definition of a proper mathematical structure to explicitly formulate the Plant model and the Noise model introduced in section 4. Here, the so-called Box-Jenkins (BJ) [18, 10] mathematical model has been adopted. It expresses mathematically both plant and noise model in terms of rational polynomials functions.

The discussion is structured as follows. In section 5.1 the BJ model is presented. Starting from the SISO simplification in Equation (9), the properties of this mathematical structures and the estimation of its parameters are delineated. In particular, it is shown how the combination of BJ and Prediction Error Methods (PEM) is used to concurrently identify both acoustic scattering and noise sources by exploiting one single excited LES. In section 5.2 the numerical settings of LES are shown.

5.1. Parametric identification

In this work, the noise and plant models introduced in section 4 are mathematically formulated by using a Box-Jenkins (BJ) mathematical structure [10]. In application to the SISO simplification in Equation (9), the BJ model can be written as

$$f_d^{BJ}[k, \Theta] = B(q, \Theta) F(q, \Theta) f_u^{LES}[k] + \frac{C(q, \Theta)}{D(q, \Theta)} e[k]$$

where $B(q, \Theta), C(q, \Theta), D(q, \Theta), F(q, \Theta)$ are polynomial functions of the parameter vector $\Theta$. Equation (17) can be explicitly expressed as

$$f_d^{BJ}[k, \Theta] = \sum_{n=0}^{N_d} b_n q^{-n} f_u^{LES}[k] + \sum_{n=0}^{N_e} c_n q^{-n} e[k]$$
where \( c_0, d_0, f_0 = 1 \) to grant identifiability of the system \([10]\). \( N_b, N_c, N_d, N_f \) represent the number of parameters and thus the orders of the filters \( B(q, \Theta) \), \( C(q, \Theta) \), \( D(q, \Theta) \), \( F(q, \Theta) \), respectively. Therefore, the vector of the unknown parameters to be identified by means of SI is the following

\[
\Theta = \{ b_0 \ldots b_{N_b}, c_1 \ldots c_{N_c}, d_1 \ldots d_{N_d}, f_1 \ldots f_{N_f} \}.
\]

The output of the BJ model is represented by the variable \( f_d^{BJ}[k, \Theta] \). The input of the model is indicated by the term \( f_d^{LES}[k] \) since it corresponds to the incoming travelling wave in the LES domain. On the one hand, the polynomials \( B(q, \Theta) \) and \( F(q, \Theta) \) describe mathematically the deterministic response (acoustic transfer coefficient) of the SISO subsystem. On the other hand, the polynomials \( C(q, \Theta) \) and \( D(q, \Theta) \) model the stochastic component (or the noise sources) of the system. The polynomials \( F(q, \Theta) \) and \( D(q, \Theta) \) are known as autoregressive terms. They relate the current state of the output \( f_d^{BJ}[k, \Theta] \) with the past \( k-N_f \) or \( k-N_f \) states of the output itself. These afford the modelling of systems whose response to an unit impulse is infinite only using a limited number of parameters.

The established LES/SI method based on Finite Impulse Response (FIR) and Wiener-Hopf inversion can be seen as a special case of Equation (17). Indeed, the FIR model can be obtained from Equation (17) by assuming \( C(q, \Theta) = D(q, \Theta) = F(q, \Theta) = 1 \). Nevertheless, as shown in \([12, 11]\), FIR does not afford a model of the noise sources.

Considering Equation (17), besides the parameters \( \Theta \) to be estimated, also the term \( e[k] \) is unknown. It represents a Gaussian white noise and therefore it is unpredictable. Consequently, Equation (17) cannot be directly used to estimate the parameters \( \Theta \). It must be before rewritten in the so-called 1-Step ahead prediction form \([10, 13]\),

\[
f_d^{BJ}[k|k-1, \hat{\Theta}] = \frac{D(q, \hat{\Theta})B(q, \hat{\Theta})}{C(q, \hat{\Theta})F(q, \hat{\Theta})} f_d^{LES}[k] + \frac{C(q, \hat{\Theta}) - D(q, \hat{\Theta})}{C(q, \hat{\Theta})} f_d^{LES}[k].
\]  

Hence, Equation (18) exploits all the known past inputs \( f_d^{LES}[k] \) and all the known past responses \( f_d^{LES}[k] \) of the system to be identified (the acoustic element simulated by LES) to predict the next step-ahead response of the model \( f_d^{BJ}[k|k-1, \Theta] \). \( \hat{\Theta} \) are the estimated parameters from the parametric SI. By using a least square error minimization on the Prediction Error \( e[k, \Theta] = (f_d^{LES}[k] - f_d^{BJ}[k|k-1, \Theta]) \) these can be estimated as

\[
\hat{\Theta} = \arg \min_{\Theta} \left\{ \sum_{k=1}^{N} (f_d^{LES}[k] - f_d^{BJ}[k|k-1, \Theta])^2 \right\}.
\]

where \( N \) is the length of the data series extracted from the LES. The SI methods based on this parametric description and on the minimization of the prediction error \( e[k, \Theta] \) are called generally Prediction Error Methods (PEM) \([10]\).

In case of the 2-Port acoustic element, considering the form of the plant and noise model in Equation (16) and the mathematical BJ structure, Equation (17) can be written as

\[
\begin{align*}
\{ f_d[k] \} & = \begin{bmatrix} B_1(q, \Theta) & B_2(q, \Theta) \\ F_1(q, \Theta) & F_2(q, \Theta) \end{bmatrix} \{ \hat{\Theta} \} + \begin{bmatrix} G_{d1}(q, \Theta) \\ G_{d2}(q, \Theta) \end{bmatrix} e[k] \\
& + \begin{bmatrix} c_{11}(q, \Theta) & c_{12}(q, \Theta) \\ c_{21}(q, \Theta) & c_{22}(q, \Theta) \end{bmatrix} \begin{bmatrix} e_1[k] \\ e_2[k] \end{bmatrix} \\
& = \begin{bmatrix} H_1(q, \Theta) \\ H_2(q, \Theta) \end{bmatrix} e[k] \\
& = \begin{bmatrix} e_1[k] \\ e_2[k] \end{bmatrix}
\end{align*}
\]  

Equation (19) is the mathematical formulation adopted in this work to estimate both noise sources and acoustic scattering matrix of the tandem diaphragm configurations considered. Here, the number of parameters to be identified has been set in an optional manner \([11, 12]\). The criterion of parsimony and the so-called Test of Whiteness \([13, 10]\) have been used. The parameters required to correctly identify the system is the minimum number of coefficients whereby the prediction error \( e[k, \hat{\Theta}] \) is a white process (white noise) uncorrelated with all the system inputs. Indeed, significant cross-correlations between inputs and \( e[k, \hat{\Theta}] \) imply a wrong characterization of the plant model. Furthermore, significant autocorrelations of the prediction error correspond to an improper modelling of the noise sources.

5.2. LES approach and settings

Large Eddy Simulations have been carried out by using the code AVBP, developed by CERFACS and IFP \(^1\). It solves the compressible Navier-Stokes equations on structured and unstructured meshes. Here, a second order in both space and time Lax-Wendroff scheme has been adopted. Higher order schemes have not been taken into account in the present work since the acoustics has to be solved only in the plane wave frequency range. This implies to consider frequencies below the cut-off frequency of the duct, i.e. \( f_{cut-off} = 6700 \) Hz. Indeed, the state of the art of the LES/SI method affords only a modelling of the aeroacoustic properties of an element below the cut-off frequency. In particular, for the specific case analysed, a frequency range \( f = [0 \ldots 4000] \) Hz has been considered.

The computational grids adopted consist in completely structured hexahedral meshes for both tandem diaphragm configurations. The values of the specific mesh parameters are reported in Table II. In the core region, between the two singularities, the elements size is of the order of the Taylor

\(^{1}\) www.cerfacs.fr
Table II. Mesh parameter details for double orifice configurations: L41 and L70.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^+$</td>
<td>4</td>
<td>Radial wall refinement</td>
</tr>
<tr>
<td>$x^+$</td>
<td>4</td>
<td>Axial wall refinement</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\sim 2.65 \times 10^{-4}$ m</td>
<td>Taylor microscale</td>
</tr>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>$8.65 \times 10^{-2}$ m</td>
<td>Min wave length</td>
</tr>
</tbody>
</table>

The grid has been furthermore refined both in axial and in radial direction in proximity of the two singularities. The wall is resolved. Therefore, the first node near the wall, at the orifice sections, is positioned radially at $r^+ = 4$ unit walls. In the duct upstream and downstream the two orifices, the mesh has been progressively coarsened in the axial direction. Nevertheless, in order to not introduce significant dissipation and dispersion errors, the maximal axial size of the elements has been determined to have at least 110 nodes within the minimum wave length of interest [9]. This corresponds to the wavelength $\lambda_{\text{max}}$ of the maximal frequency considered $f_{\text{max}} = 4000$ Hz.

The turbulent eddies bigger than the size of the mesh elements are resolved, whereas the smaller ones are modelled by means of a subgrid scale model. Here, the Wall Adapting Local Eddy viscosity model (WALE) has been used [20]. This has been chosen for its better capacity to reproduce the turbulent statistics of the subgrid scales near the wall.

Simulations have been carried out for a Courant-Friedrichs-Lewy number CFL = 0.7. Completely non-reflecting boundary conditions have been adopted [21, 22]. These consist in a modification of the so-called Navier-Stokes Characteristic Boundary Conditions (NSCBCs) [23] based on the Plane Wave Masking (PWM) technique [24]. Hereby, the acoustic travelling wave outgoing the configuration of interest is “masked” in order to not be reflected back in the numerical domain. The use of these boundary conditions yields robustness of the numerical estimations obtained by the LES/SI method. Moreover, it avoids resonant conditions due to spurious numerical reflections at the boundaries.

Two separate LES with and without external acoustic excitation have been carried out. On the one hand, a non-excited LES has been used to directly compare the noise sources assessed experimentally with the ones simulated numerically. On the other hand, an LES with a broadband acoustic excitation has been performed to estimate both Noise model and Plant model (or scattering matrix) of the configurations. Here, a wavelet type broadband signal with constant PSD in the range of frequency of interest has been used [25]. This kind of signal affords two uncorrelated inputs (ingoing acoustic waves), yielding a better estimation of the models through SI. The amplitude of the signal has been fixed to 1.7% of the $U_{\text{bulk}}$ to not introduce non-linear response of the shear layers developed at the orifice edges.

Acoustic data series are extracted from the LES at the inlet and at the outlet of the computational domain. Here, the pressure and acoustic fluctuations are assessed at different planes positioned in the upstream and downstream duct. Subsequently, the characteristic acoustic waves are retrieved from the flow local velocity $u$ and pressure $p$ by means of the so-called Characteristic Based Filter (CBF) [26].

6. Results

In this section, the results obtained from the parametric identification presented in section 5.1 are compared to experimental measurements. The latter have been obtained by measuring, on the same geometry with the same flow, the scattering matrix by a two sources method and the noise sources without external excitation.

The experimental test rig used has been described in Testud et al. [14]. 4 microphones upstream and 4 microphones downstream the test section have been employed. Robustness is acquired by solving an overdetermined system of equations describing the phase delays due to the acoustic wave propagation between microphones. Here, the so-called Moore-Penrose pseudo-matrix inversion [3] is performed to solve the system. Noise sources have been measured by means of a full source characterization [3, 2]. Reflections at the boundaries as well as the scattering across the elements have been taken into account to correct the experimental power spectral densities of the outgoing pressure waves.

Results are shown in the normalized frequency or Strouhal number, defined as follows

$$St = \frac{ft}{U_d} \quad \text{with} \quad U_d = U_{\text{bulk}} \left(\frac{D}{d}\right)^2,$$

where $f$ indicates the frequency. Both acoustic scattering and noise sources are investigated. Moreover, the interaction between the two orifices in a duct is analysed through the so-called whistling criterion.

6.1. Acoustic scattering

The magnitude and the phase of the identified acoustic scattering matrix are compared against experimental results in Figures 5a, 5b and in Figures 6a, 6b for the configuration L41 and L70, respectively.

Numerical and experimental results are in good accordance. The magnitude minima and maxima are found at the same Strouhal number as in the experiments. Moreover, also the phase shifts of the scattered waves are well predicted.

Noticeable deviations between numerical and experimental results are observed for the magnitude of the transmission coefficient $T_{\text{tot}}$ and for the reflection coefficient $R_{\text{tot}}$ of the two configurations. Here, the intense turbulence in the downstream duct plays a significant role for the accuracy of the identification. Nevertheless, the highest disagreement are observed at low St. This suggests a lack of resolution in the low frequency range. Longer data series would increase the frequency resolution, yielding a better
characterization of the low frequency range. Here, the experimental measurements have a frequency resolution of 1 Hz, whereas the numerical identification is carried out with a frequency resolution of 3.33 Hz (St = $4.625 \times 10^{-4}$) and 4.54 Hz (St = $6.305 \times 10^{-4}$), for the case L41 and L70, respectively. The former corresponds to a time series of length $T = 0.3 \text{ s}$. The latter refers to a data length of $T = 0.22 \text{ s}$. These data lengths have been adopted on the one hand to keep the time costs of the simulations affordable and to have small variances in the estimated parameters, on the other.

Increasing the time length of the data series available would potentially improve the accuracy of the identification results. The higher the amount of data available the lower the variance of the parameters identified [10]. Small variations in the SI results for different lengths of time series are therefore expected. Convergence of SI results may be expected only asymptotically, for an infinite length of data samples. This is not feasible in practical applications. Nevertheless, the simple increment of the data series available does not automatically imply a better agreement with the experiments. This depends on several factor such as the LES model adopted and the model structure used in SI.

Significant levels of disagreement are found for the configuration L41 around $St = [0.2 \text{ to } 0.3]$. As it will be shown in section 6.3, the experimental power spectral densities of the noise for this configuration is characterized by a significant whistling in this frequency range. The presence of a noticeable tonality in the noise spectra could significantly influence the reliability of the measurements. Indeed, in that frequency range, the experimental assessment is carried out by using a lower Signal to Noise Ratio (SNR). Consequently, a lower accuracy is expected. Nevertheless, an increment of the level of the excitation amplitude could introduce non-linear interactions between the acoustic excitation and the shear layers developed at the orifice plate, invalidating the assumption of linear acoustic.

6.2. Whistling criterion

The acoustic scattering matrix assessed in section 6.1 is used here to estimate the whistling potentiality of each tandem orifice configuration. Identified models are compared against experimental results in Figure 7a and Figure 7b for the case L41 and L70, respectively. Experiments and numerical SI models are in a good agreement. The minimum values of the eigenvalue $\xi_{\text{min}}$ computed from the identified model are quite well predicted and are found at the same Strouhal number as in the experiments. The identified model in Figure 7a has a second whistling frequency at $St \approx 0.2$ which is not present in the experiments. This is most probably due to the limited number of data samples available yielding an improper estimation in the low Strouhal range.
The identified noise sources are compared in terms of Power Spectral Density (PSD) against the experiments in Figure 9 and Figure 10, for the L41 and L70 configuration, respectively. Here, a further non-excited LES has been used to directly compute the noise PSD without any additional modelling. Hereby, it is possible to directly compare the experimental assessments against the noise sources simulated by means of LES.

The broadband component of the noise sources is well simulated by the LES. The decays in the PSD at high St of the LES results are in accordance with the experiments for both configurations. Noticeable deviations are observed at low frequencies. Here, the experimental PSDs present values around $4 - 5$ dB/St higher than the numerical ones. Moreover, the tonal noises measured in the configuration L41 at St = 0.263 and at St = 0.526 are not observed in the non-excited LES.

These differences are due to both experimental and numerical issues. Indeed, the LES has been carried out with perfectly non-reflecting boundary conditions. However, though in the experiments anechoic terminations have been adopted, partial reflections are present. The acoustic feedback introduced by those, in case of configurations of high whistling potentiality as L41, can generate resonant acoustic modes. These can be driven by the internal flow has a higher whistling potentiality than each single orifice composing the system. Hence, the acoustic interactions between the two singularities increase the overall power generation of the system. In contrary, L70 produces less acoustic power than the individual orifice plate. Consequently, the whole configuration is more acoustically dissipative than one single orifice.

The acoustic feedback between the two singularities can be the reason for an increment or a decrement of power generation compared to the one observed in the single orifice case. The acoustic reflections taking place at the two singularities determine a so-called closed-loop system. Hereby, the acoustic waves transmitted across the upstream orifice are partially reflected by the downstream singularity in the upstream direction. These, once reached the upstream diaphragm, are then partially reflected in the downstream direction, yielding a feedback acoustic wave at the upstream singularity. If, at a given frequency, this feedback acoustic wave is in phase with the transmitted wave across the upstream diaphragm, a resonant mechanism is established. This depends only on the phase difference between transmitted acoustic wave across the upstream orifice and feedback wave due to the feedback loop. Those phases depend on the duct length between the singularities and on the phases shifts introduced by the reflections at each orifice. If a resonant condition takes place in the Strouhal range of power generation of each orifice $St = 0.2 - 0.4$ more acoustic power can be generated by the whole system. The same considerations can be done by symmetry for the acoustic feedback at the downstream singularity.

6.3. Noise sources

Comparing Figure 7a against Figure 7b, the configuration L41 has a higher whistling potentiality than L70. In other words, the tandem orifice whereby the orifices are closer is more prone to whistle when a proper acoustic feedback with the surrounding system is established.

In Figure 8, the whistling criterion for one single orifice composing the double orifice systems is displayed. Experimental measurements have been performed by Testud [14]. The numerical model has been identified by Sovardi et al. [11] by using the same parametric LES/SI approach used here for the tandem orifices. Both numerical and experimental results predict a frequency range of power generation for $St = [0.2 - 0.4]$. Nevertheless, as pointed out in [11], the identified model is slightly shifted towards lower frequencies.

By comparing Figure 7 against Figure 8 some important considerations can be stated. The configuration L41

![Figure 7. Whistling Criterion. Grey x marker: Experimental results from LAUM. Black continuous line: Identified acoustic scattering from SI and a BJ model.](image1)

![Figure 8. Whistling criterion of one single orifice. Grey x marker: experimental results from Testud [14]. Black continuous line: Identified model SI model with BJ model.](image2)
field to high amplitude narrow banded acoustic fluctuations and consequently to whistling conditions. Though, a full source characterization [3] has been performed, the amplitude of the whistling is given by non-linear phenomena. Thus, the effects of reflections can be difficult to remove by a linear process when whistling is present.

Another difference between experiments and LES is at the inlet boundary condition. The inflow in the LES is laminar, while it is a fully developed turbulent flow in the experiments. Preliminary LES analysis have been carried out on the configuration L41 imposing homogeneous isotropic turbulent fluctuations with different amplitudes at the inflow. This has been done to verify a possible relation between a turbulent inlet and the tonality observed in the PSD of the noise. Nevertheless, no significant effect has been observed.

As already observed from the comparison between Figure 7a and Figure 7b, the case L41 presents higher whistling potentiality than L70. Therefore, by using the same acoustic terminations in the experimental set-up, partial reflections at boundaries induce more easily self-sustained acoustic oscillations in the L41 case than in L70.

Further reasons for these discrepancies could be the mesh refinement or the modelling of the SGS scales adopted in the LES. However, besides the LES settings used in this work (wall refined hexahedral grid with WALE), several other numerical options for LES have been checked. These include both structured and unstructured (tetrahedral) meshes as well as different SGS models, i.e. static and dynamic Smagorinsky. Nevertheless, no significant changes in the noise predicted by LES have been observed.

The identified noise models are in perfect agreement with the PSD of the noise sources educed from the non-excited LES for both L41 and L70 configurations. The PSD given in the plot is smooth because it is the result of feeding the noise model with a GWN time series of practically infinite length. The variance observed in the PSD of the noise sources extracted from the non-excited LES is due to the limited length of the simulations performed. In other words, the PSDs of the noise sources for a non-excited LES would asymptotically converge to the smooth noise model, when an infinite simulation time is considered.

This confirms the accuracy of the SI approach. Since the identification is performed on data series extracted from an excited LES, the best results achievable should resemble the PSDs obtained from the LES without external excitation. Therefore, the noise model predicted for the L41 case does not present any tonality in the noise power spectral densities.

In order to achieve a better agreement between the noise models and the experiments, improvements in the LES modelling should be adopted. In the ideal case, the same acoustic impedance of the experimental terminations together with realistic turbulent statistics at the inflow must be used as boundary conditions for the simulations.

7. Conclusions

An innovative parametric LES/SI approach has been used to concurrently characterize both acoustic scattering and noise sources of two tandem orifice configurations. The identified models have been extensively validated against the experimental assessment carried out at the Laboratoire d’Acoustique de l’Université du Maine (LAUM). Specifically, numerical and experimental results are compared in terms of acoustic scattering matrix, whistling potentiality and powers spectral density of the noise sources. The accuracy of the models identified by means of LES/SI strictly depends on the LES quality. Only the acoustic properties reproducible through an LES are indeed identifiable.

The acoustic interactions between the two singularities composing the tandem orifice configurations have been analysed by using the so-called whistling criterion. The whistling potentiality of the ducted systems depends on the distance between the two orifices. Here, the acoustic feedback between the two singularities is responsible for the amplification of the acoustic power generated at each orifice.

The broadband component of the noise sources has been well captured and modelled. Tonal noise sources observed
in the experiments have not been captured by the LES. This is most probably due to partial acoustic reflections at the terminations of the experimental set-up. Indeed, acoustic duct systems with high whistling potentiality are prone to whistle when a proper acoustic feedback due to reflections is established.

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