Whistling of an orifice in a reverberating duct at low Mach number

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An experimental investigation of the parameters controlling the whistling frequency and amplitude of an orifice in a confined turbulent flow is undertaken. A circular single hole orifice with sharp edges, a hole diameter equal to 0.015 m and a thickness equal to 0.005 m, is arranged in an air test rig with an inner diameter equal to 0.03 m. The Mach number ranges around 0.02 and the Reynolds number around $10^5$. Variable reflecting boundary conditions are arranged upstream and downstream, and several flow velocities are tested. It is found that the Bode–Nyquist criterion accurately predicts the conditions of self-sustained oscillation and the value of the whistling frequency. Furthermore, it is found that the acoustic velocity in whistling regime varies from 1% to 15% of the steady flow velocity, and that it depends on the overall acoustic reflection of the surrounding pipe and on the Strouhal number.

I. INTRODUCTION

Single hole orifices are known to sometimes generate single tone noise when adequate conditions are met. A detailed description of vortex shedding with lock-in can be found in classical literature, and in papers dedicated to acoustically induced whistling phenomena. The whistling of an orifice in a confined flow with acoustical reflecting conditions has been investigated by Anderson, Testud et al., and Karthik et al. The physical mechanism generally proposed is that the resonant acoustic field modulates the unstable shear layers where the downstream flow separates, so that energy is transferred from the flow to acoustics.

A criterion of whistling ability has been proposed by Starobinski and Aurégan and experimentally validated by Testud et al. for thin sharp edge orifices. This criterion is based on an acoustic power balance expressed with the help of the scattering matrix

$$
\begin{pmatrix}
    p_d^+ \\
    p_u^+
\end{pmatrix}
= \begin{pmatrix}
    T^+ & R^- \\
    R^+ & T^-
\end{pmatrix}
\begin{pmatrix}
    p_u^- \\
    p_d^-
\end{pmatrix},
$$

where the complex transmission and reflection coefficients $T^+$, $T^-$, $R^+$, and $R^-$ are functions of the frequency and of the fluid velocity, where $p_u^+$ and $p_d^-$ are the incident propagating pressure waves, and where $p_u^-$ and $p_d^+$ are the scattered propagating pressure waves. The measurement of the matrix elements can be achieved with the two sources method.

The criterion indicates that an orifice is prone to whistling if there is a range of frequencies for which the scattered acoustic power can exceed the incident acoustic power. In other words, such an orifice behaves as an amplifier of acoustic pressure waves, and instability occurs if the surrounding duct exhibits a sufficient feedback. Once the instability is triggered, an initial perturbation undergoes an exponential growth, up to the point where nonlinear effects stabilize the oscillation to a steady state harmonic regime. Such a process can be described by a single degree of freedom oscillator, with a nonlinear acoustic gain at the orifice, and a linear acoustic response of the surrounding duct.

Previous studies have shown that some orifices generate pure tone noises in the presence of acoustic reverberating conditions, and that they do not in the presence of acoustic absorbing conditions. The current study deals with the evaluation of the influence of the acoustic surrounding on the onset of self-sustained oscillations on the one hand, and on the amplitude of the tone noise in stabilized whistling regime on the other hand. In the latter case, the study is focused on the whistling frequency and on the pure tone amplitude as a function of an overall acoustic reflection coefficient. Based on previous studies, it was chosen to test a straight edge orifice. The analysis is similar to the one of Mast and Pierce for flow excited resonators and to the one of Graf and Ziada for side-branches arrangement.

In Sec. II, a theoretical description of the different phenomena responsible for pure tone generation is given. An expression of the acoustic gain of the orifice and of the acoustic losses of the surrounding is proposed, which is used to predict the conditions of self-sustained oscillations using the Bode–Nyquist criterion. Subsequently, the steady whistling regime is described with the help of a balance of the saturated acoustic gain of the orifice and of the acoustic losses of the surrounding.

The experimental setup is then described. Special boundary conditions are arranged, such that the amplitude...
and the argument of the overall reflection coefficients can be tuned. For that purpose, on the upstream side, either an expansion chamber with a reflection coefficient close to unity or a shallow cavity with a variable depth can be arranged. On the downstream side, an open end pipe is arranged, to which a perforated pipe and absorbing foam disks can be added.

Results are provided in Sec. IV: The Bode–Nyquist criterion is shown to accurately predict the occurrence of self-sustained oscillations. The frequency associated with the onset of instability determined by the linear theory is found to be very close to the whistling frequency in steady state regime. Furthermore, it is shown that both the modulus of the overall acoustic reflection and the Strouhal number have a significant influence on the amplitude of the pure tone noise.

II. THEORETICAL BACKGROUND

The flow–acoustic interaction in the shear layer downstream of the orifice converts incident pressure waves $p_u^+$ and $p_d^+$ in scattered pressure waves $p_u^-$ and $p_d^-$. Equation (1) describes this process in the framework of small perturbations. Such a description is correct from an aeroacoustic point of view, but it can be simplified at low Mach numbers. In the latter case, the fluid compressibility inside the orifice does not play a significant role, and the four elements of the scattering matrix are no longer independent.\textsuperscript{20–22} Benefiting from this circumstance, a new set of variables can be introduced, which facilitates the physical understanding of acoustic amplification and of pure tone generation.

Basically, the idea consists of distinguishing “velocity excitations” of the orifice, associated with $p_u^+ = -p_d^+$, from “pressure excitations,” associated with $p_u^+ = p_d^-$. In the first case, the interaction of the acoustic waves with the shear flow has been reported to be maximum.\textsuperscript{10} In the second case, an equal increase of the pressure on both sides of the orifice should not generate acoustic amplification or dissipation. This property is known to be related to the fact that at low Mach numbers, the orifice area can be considered as almost acoustically compact, so that the upstream and downstream acoustic velocities are approximately equal.\textsuperscript{20–22}

From now on, the sums and the differences of the incident and of the scattered propagating pressures shall be used as a new set of variables, keeping in mind that the phenomena of interest are essentially due to the pressure difference terms.

A. Acoustic amplification in linear regime

Using the new set of variables, the scattering matrix of Eq. (1) can be rewritten after a straightforward calculation as

\[
\begin{pmatrix}
    p_u^+ + p_d^- \\
    p_d^+ - p_u^- \\
\end{pmatrix} = \frac{\Sigma T + \Sigma R \Delta T + \Delta R}{\Delta T - \Delta R \Sigma T - \Sigma R} \begin{pmatrix}
    p_u^+ + p_d^- \\
    p_u^- - p_d^+ \\
\end{pmatrix},
\]

where the new transmission and reflection coefficients are functions of the sums and differences of the former ones, $\Sigma T = (T^+ + T^-)/2$, $\Sigma R = (R^+ + R^-)/2$, $\Delta T = (T^+ - T^-)/2$, and $\Delta R = (R^+ - R^-)/2$. In order to determine the elements of the scattering matrix, experiments were carried out with the same test rig and the same procedure as in Testud \textit{et al.},\textsuperscript{10} so that details are not provided here. It should be noted that the linear regime is ensured by keeping the dimensionless acoustic velocity $\nu/U$ smaller than 0.03. The resulting transmission and reflection coefficients are reproduced in Fig. 1, as a function of a Strouhal number $St = ft/U_{hole}$, based on the orifice thickness $t$ and on the velocity $U_{hole} = UD^2/d^2$ in the orifice hole, $d$ being the hole diameter and $D$ the pipe diameter.

As expected, the diagonal element corresponding to a pressure excitation is close to unity, and the modulus of the extradiagonal elements are close to zero. The arguments of the extradiagonal elements have not been plotted, because their modulus are smaller than 0.1. The transmission and reflection coefficients of the original scattering matrix appear to reasonably fulfill the low Mach number relations,\textsuperscript{20–22} namely

\[
T^+ = T^- + 1 = 1 - R^+ = 1 - R^-.
\]

The acoustic behavior of the orifice can hence be described by the simplified scattering matrix

\[
\begin{array}{c}
\text{St} = ft/U_{hole} \\
\text{module} \\
\text{argument} \\
\end{array}
\]

FIG. 1. Modulus (a) and argument (b) of the scattering matrix elements in Eq. (2). Solid lines: $\Sigma T - \Sigma R$, dashed lines: $\Sigma T + \Sigma R$, dotted and dashed-dotted lines: $\Delta T \pm \Delta R$. Black: $U = 6$ m s$^{-1}$, dark gray: $U = 9$ m s$^{-1}$, and light gray: $U = 12$ m s$^{-1}$.
\[
\begin{pmatrix}
p_d^+ + p_u^- \\
p_d^- - p_u^-
\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{G} \end{pmatrix} \begin{pmatrix} p_d^+ + p_u^- \\ p_d^- - p_u^- \end{pmatrix},
\]

where the acoustic gain \( \mathcal{G} \) fully describes the mechanism of flow–acoustic interaction at low Mach numbers. This representation overlooks the local influence of the compressibility, but it greatly simplifies the analysis of experimental data by reducing the acoustic behavior of the orifice to only one parameter.

The use of the acoustic gain \( \mathcal{G} \) brings out, in a straightforward manner, trends observed in previous studies; the equivalent reflection coefficients at the location of the orifice and backward propagating wavenumbers where \( \mathcal{L}_u \) and \( \mathcal{L}_d \) have shown, respectively, that the onset of thermoacoustic instability of a combustor and whistling in T-junction could be studied with the help of the Bode–Nyquist criterion. Applying this procedure to the current study, the loci in the complex plane of the product \( -\mathcal{G}F \) for real frequencies are plotted, and, if the curve encircles the critical point \((-1, 0)\), self-sustained oscillations occur. More details about the Bode–Nyquist criterion can be found in the aforementioned papers. Its application is illustrated in Fig. 2 for a whistling and a nonwhistling test. Details of the experiments are provided in Sec. III. It is sufficient here to mention that the two cases were obtained for a flow velocity equal to \( 8 \text{ m s}^{-1} \). The black curve corresponds to a perforated pipe downstream and the gray curve corresponds to the same perforated pipe with two foam disks.

For practical purposes, the unstable frequency is defined here as the one that makes the imaginary part of \( -\mathcal{G}F \) vanish. In the case illustrated in Fig. 2, the unstable frequency is equal to about 2050 Hz.

It is worth highlighting the fact that the procedure described here is performed in the linear domain, so that the stability of the system and the unstable frequencies can be determined, but the steady state regime cannot. When the steady whistling regime is reached, nonlinear effects have occurred, which tend to lower the equivalent acoustic gain of the orifice, as discussed in the following.

D. Acoustic balance in whistling regime

In steady whistling regime, it is assumed that the analysis can be focused on the fundamental frequency by introducing an equivalent acoustic gain. Such a description overlooks the nonlinear nature of acoustic amplification, and the role played by harmonics of the fundamental frequency. As the harmonic pressure peaks measured were always lower by at least 1 order of magnitude than the fundamental peak, the nonlinear gain representation is assumed to hold without further proof. Then let the steady state regime be associated with a saturated value \( \mathcal{G}_{sat} \) of the acoustic gain, according to

\[
\begin{pmatrix}
p_d^+ + p_u^- \\
p_d^- - p_u^-
\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{G}_{sat} \end{pmatrix} \begin{pmatrix} p_d^+ + p_u^- \\ p_d^- - p_u^- \end{pmatrix}.
\]

C. Linear stability analysis

Sattelmayer and Polifke\textsuperscript{23} and Karlsson and Åbom\textsuperscript{24} have shown, respectively, that the onset of thermoacoustic

![FIG. 2. Nyquist diagrams of a nonwhistling case (gray curve) and of a whistling case (black curve). Successive plus markers are spaced by steps of 50 Hz.](image)
This situation occurs whenever nonlinear effects inside the orifice adapt the acoustic gain to the acoustic feedback of the surrounding pipe. In other words, the steady state gain cannot be considered any longer as an intrinsic property of the orifice, it depends on the amplitude of the whistling.

An illustration of this proposition is given by the scattering matrix elements in Fig. 3, which were obtained through the same two-sources method as in Sec. II A, but with a higher amplitude of the pressure generated by the loudspeaker. The experiment was not designed to accurately control the amplitude of the acoustic velocity, but only to qualitatively assess its influence; be it enough to mention that the dimensionless acoustic velocity $u/U$ ranged from 0.04 to 0.32, the linear regime being ensured for values up to 0.03. The values obtained show that the simplified matrix proposed in Eq. (10) is appropriate; an equal increase of the pressure on both sides of the orifice does not generate noticeable effects, the extradiagonal elements can be neglected, and the analysis can still be based on the pressure differences $p_d^+ - p_a^-$ and $p_a^+ - p_d^-$. The comparison with the matrix elements of Fig. 1 shows that an increase in the excitation amplitude leads to a decrease of the acoustic gain modulus in the Strouhal range from 0.2 to 0.4, associated with a smoothing of its argument.

Combining Eqs. (10) and (8), the balance of acoustic amplification and feedback can be written

$$G_{\text{sat}} F = 1. \quad (11)$$

The balance of gain and feedback described by Eq. (11) has the classical form of linear oscillator studies with a single degree of freedom. The feedback term $F$ is known to be a key parameter for the onset of self-sustained oscillations, and in the following, its value is provided with experimental data, in order to evaluate its influence on the whistling amplitude.

III. EXPERIMENTAL PROCEDURE

The test rig consists of a straight circular pipe in ambient air, with an inner diameter of 0.03 m. It can be operated at flow velocities of the order of 10 m s$^{-1}$, i.e., at Mach numbers of the order of 0.01 and at Reynolds numbers of the order of $10^4$. As the orifice thickness is equal to 0.005 m and as the hole diameter is equal to 0.015 m, the fluid velocity inside the hole is equal to about 40 m s$^{-1}$. For a Strouhal number equal to about 0.3, the expected whistling frequency is of the order of 2000 Hz, which is lower than a third of the cut-off frequency of nonplanar acoustics.

The design of the pipe lengths upstream and downstream of the orifice results from a compromise between the minimum length required for the turbulent jet influence to vanish and the need to keep the total length of the acoustic system as short as possible, so that it does not exceed a few wavelengths. This last requirement is due to the fact that a large number of wavelengths in the system increases the uncertainties of acoustic identification. The choice of short pipe simplifies the analysis of the parameters controlling the whistling features, especially for the frequency. The choice is made of an upstream length of the order of 0.1–0.2 m, and of a downstream length of the order of 0.2–0.3 m. As a consequence, only one microphone on each side of the orifice can be arranged.

Upstream and downstream, acoustic boundaries have to be sufficiently reflecting to enhance self-sustained oscillations. Furthermore, these acoustic boundaries have to be designed in such a way that the amplitude of $F$ and the whistling frequency can be easily modified.

A. Experimental setup

A scheme of the test rig is presented in Fig. 4. As mentioned previously, it is an air test rig with an inner diameter equal to 0.03 m and a thickness equal to 0.004 m. Several supports are arranged all along it to avoid vibrations. A compressor Aerzen Delta blower GM10S generates a constant flow rate from 0.03 to 0.15 m$^3$ s$^{-1}$, corresponding to a flow velocity inside the pipe varying from 6 to 12 m s$^{-1}$. It is measured by a turbine flow meter ITT Barton 7402 provides an accuracy of $10^{-1}$ m s$^{-1}$. The duct temperature is measured by a transducer with an accuracy of $\pm 0.1$ K, which is used to estimate the speed of sound and the visco-thermal losses in the pipe. In order to ensure low reflection conditions from the

![Figure 3](image-url)
flow meter and the compressor, an anechoic termination is mounted upstream of the test section.

Fluctuating pressures are measured upstream and downstream of the orifice with 1/4 in. B&K 4938 microphones. Each microphone is connected to a preamplifier B&K Nexus 2670, and the output signals are sent to an acquisition system HP 3565. In steady state conditions, power spectra are calculated by averaging 150 samples, using Hanning windows with an overlap of 50%. The sampling frequency is equal to 16384 Hz, the measuring time is equal to about 3 min, and the frequency step is equal to 4 Hz. Using the spectra provided by the acquisition system, the amplitude of the pressure peaks at the whistling frequency $f_w$ is obtained by taking the square root of the summation of the spectra in the range $f_w \pm 4$ Hz. This procedure was checked by applying a calibrated harmonic signal to the acquisition system; the estimated amplitude matched the actual value with an accuracy better than 1%. To avoid any confusion in the following, the zero-peak amplitudes of the fluctuating pressure and of the acoustic velocity in harmonic regime are denoted by lowercase letters $p$ and $u$, the physical units of which are Pa and m s$^{-1}$, respectively. The power spectra are labeled with capital letters and a tilde, e.g., $\tilde{P}$ and $\tilde{U}$, and their physical units are Pa$^2$ and the m$^2$ s$^{-2}$, respectively.

B. Reflections at the boundaries

On the downstream side, an open end pipe is first tested, and a perforated pipe with a length equal to 0.0845 m is added afterwards, which generate acoustic losses. One or two foam disks can be arranged around the perforated pipe, which increase the acoustic losses. The reflection coefficients are measured using the same experimental procedure as for the scattering matrices, and the results are plotted in Fig. 5. It can be seen that the modulus of the reflection coefficient is in the range 0.4–0.8, and that its argument is almost constant for the three cases with the perforated pipe. Besides, tests performed with different flow velocities showed that this last parameter has no significant influence.

Upstream, a shallow cavity with a variable depth, $d_{cav}$, and a diameter identical to the one of the main pipe is first arranged. Its reflection coefficient is plotted in Fig. 6 for three different values of $d_{cav}$. The peaks correspond to the resonance frequencies of the cavity, as is made evident by introducing the dimensionless frequency to $fd_{cav}/c$. As in the case of the downstream arrangement, tests showed that the flow velocity has no significant influence on the reflection coefficient. This device can hence be used to modify the overall acoustic reflection coefficient by adjusting the peak frequency of Fig. 6.

It is worth mentioning that although shallow cavities are known to potentially generate tonal noise, this is not the case here. First, measurements have been done for different flow velocities without the orifice, and no sharp peak was observed. Second, such a shallow cavity can only couple with nonplanar acoustic modes of the main pipe, the first cut-off frequency of which is equal to about 6700 Hz, far beyond the range of frequencies of the study.

An expansion chamber filled with an absorbing foam can also be arranged upstream. Its reflection coefficient is plotted in Fig. 7. Once again, the flow velocity has no noticeable effect on the acoustic behavior. This device is used to generate a high reflection coefficient.

The boundary conditions, their location, and the mean flow velocities in the main pipe are given in Table I for all test arrangements.

C. Acoustic identification

As proposed in Sec. II, the analysis of the experiments is based on the amplitude of the pressure pulsations, using the acoustic feedback $F$ as a parameter. Estimations of the amplitude of the whistling and of the acoustic feedback are then required. For the sake of convenience, it is proposed that the amplitude of the whistling be expressed as the ratio...
of the acoustic velocity amplitude \( u \) to the steady flow velocity \( U \), as is currently done in experiments with shallow cavities.\(^{29,30} \) Another dimensionless number that could have been used is the ratio of the acoustic pressure difference to the dynamic pressure \( q U^2/2 \), which would have led to similar results.

The first step of the data processing deals with the estimation of the acoustic velocity amplitude at the orifice. The transfer function relating the pressure measured at the upstream sensor location to the acoustic velocity at the orifice in harmonic regime can be written in dimensionless form by expanding the pressure in a forward propagating term and a backward propagating term,

\[
H_u = \frac{(R_u - 1) e^{j k^+ L_u^m}}{1 + R_u e^{j (k^+ + k^-) L_u^m}},
\]

where \( L_u^m \) is the distance from the orifice to the upstream pressure sensor, where the expression of the wavenumbers \( k^+ \) and \( k^- \) include visco-thermal losses,\(^{31} \) and where the reflection coefficient \( R_u \) can be found in Fig. 6 or Fig. 7. In the formalism of power spectra, the dimensionless relation between the acoustic velocity spectrum at the orifice \( \tilde{U}_u \) and the measured upstream pressure spectrum \( \tilde{P}^m_u \) is

\[
\tilde{U}_u \frac{U^2}{2} = |H_u|^2 \frac{\tilde{P}^m_u}{\rho^2 c^2 U^2}.
\]

Similar expressions can be derived on the downstream side. The transfer function is

\[
H_d = \frac{(1 - R_d) e^{j k^- L_d^m}}{1 + R_d e^{j (k^+ + k^-) L_d^m}},
\]

where \( L_d^m \) is the distance from the orifice to the downstream pressure sensor, and where the reflection coefficient \( R_d \) can be found in Fig. 5. The acoustic velocity spectrum at the orifice \( \tilde{U}_d \) is related to the measured downstream pressure spectrum \( \tilde{P}^m_d \) by

\[
\tilde{U}_d \frac{U^2}{2} = |H_d|^2 \frac{\tilde{P}^m_d}{\rho^2 c^2 U^2}.
\]

A comparison of the two estimations of the acoustic velocity power spectrum is given in Fig. 8(a), corresponding to the

FIG. 6. Reflection coefficient modulus (a) and argument (b) at the shallow cavity for different depths. \( M = 2.6 \times 10^{-2} \) (solid line); \( d_{cav} = 3.7 \times 10^{-2} \) m (dashed-dotted line); and \( d_{cav} = 4.7 \times 10^{-2} \) m (dotted line).

FIG. 7. Reflection coefficient modulus (a) and argument (b) at the expansion chamber for two Mach numbers. \( M = 2.6 \times 10^{-2} \) (solid line) and \( M = 3.4 \times 10^{-2} \) (dashed-dotted line).
arrangement numbered 1 in Table I, with a flow velocity equal to 9 m s\(^{-1}\). A sharp peak at 1900 Hz and an harmonic at 3800 Hz are observed, which constitute the meaningful part of the signal. Besides, the broadband noise generated by the turbulence of the orifice jet excites the acoustic resonances of the pipe. As can be seen, the two estimations of the acoustic velocities’ power spectra are very close, the discrepancies being mostly due to the vanishing of either \(T_u\) or \(T_d\) at certain frequencies, as illustrated in Fig. 8(b). This observation is consistent with the fact that the orifice behaves in an almost incompressible manner in the range of Mach numbers considered.

The zero-peak amplitude \(u\) of the acoustic velocity at the whistling frequency can be deduced from the power spectra with the help of the procedure described in Eqs. (13) and (15). In the cases illustrated in Figs. 8(a) and 8(b), the dimensionless acoustic velocity is estimated to 0.098 upstream and to 0.083 downstream, which constitutes a reasonably fair agreement in the framework of acoustic identification. This agreement results from the fact that both transfer functions are not close to zero and do not exhibit high slopes at the whistling frequency.

The last step of the acoustic identification deals with the estimation of the acoustic feedback \(F\) according to Eq. (11). The equivalent reflection coefficients \(R_u\) and \(R_d\) in this equation are deduced from the ones plotted in Figs. 5–7, and from Eqs. (5) and (6), which exhibit terms of the type \(e^{jkL}\). As is the case for the transfer functions \(H_u\) and \(H_d\), the combination of these exponential terms with the ratio of Eq. (11) generates a series of maxima and minima, which are responsible for uncertainties in the estimation of \(F\). Parametric studies on the lengths, the wavenumbers, and the reflection coefficients have shown that the overall uncertainty on the modulus of \(F\) can be estimated to about \(\pm 0.1\).

Finally, based on the shapes of the transfer functions \(H_u\) and \(H_d\) at the whistling frequency, a selection process of the experimental data is elaborated. Measurements for which both transfer functions are close to zero or which exhibit high slopes are removed, and the acoustic velocity is taken equal to the average of both estimations if neither of them is close to zero. A second elimination process is applied, which consists of removing the cases where the power spectra exhibit several narrow peaks, because this situation is probably due to an intermittent regime of vortex shedding, switching from one acoustic mode to the other. In that case, the amplitude of each peak may be significantly underestimated by comparison with a situation where only one peak would be present. The arrangements of Table I result from the application of these criteria.

### IV. RESULTS

#### A. Whistling frequency and instability frequency

The experiments show that whistling frequencies are locked on successive acoustic modes, in a manner similar to

<table>
<thead>
<tr>
<th>Arrangement number</th>
<th>Upstream</th>
<th>Downstream</th>
<th>(L_u) (m)</th>
<th>(L_m^u) (m)</th>
<th>(L_d) (m)</th>
<th>(L_m^d) (m)</th>
<th>(U) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Expansion chamber</td>
<td>Open pipe</td>
<td>0.222</td>
<td>0.077</td>
<td>0.260</td>
<td>0.178</td>
<td>6–12</td>
</tr>
<tr>
<td>2</td>
<td>Expansion chamber</td>
<td>Perforated pipe</td>
<td>0.222</td>
<td>0.077</td>
<td>0.260</td>
<td>0.178</td>
<td>6–12</td>
</tr>
<tr>
<td>3</td>
<td>Expansion chamber</td>
<td>Perforated pipe + 1 foam disk</td>
<td>0.222</td>
<td>0.077</td>
<td>0.260</td>
<td>0.178</td>
<td>6–12</td>
</tr>
<tr>
<td>4</td>
<td>Expansion chamber</td>
<td>Perforated pipe + 2 foam disks</td>
<td>0.222</td>
<td>0.077</td>
<td>0.260</td>
<td>0.178</td>
<td>6–12</td>
</tr>
<tr>
<td>5</td>
<td>Shallow cavity</td>
<td>Open pipe</td>
<td>0.115</td>
<td>0.077</td>
<td>0.185</td>
<td>0.093</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>Shallow cavity</td>
<td>Perforated pipe</td>
<td>0.115</td>
<td>0.077</td>
<td>0.185</td>
<td>0.093</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>Shallow cavity</td>
<td>Perforated pipe + 1 foam disk</td>
<td>0.115</td>
<td>0.077</td>
<td>0.185</td>
<td>0.093</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>Shallow cavity</td>
<td>Open pipe</td>
<td>0.115</td>
<td>0.077</td>
<td>0.185</td>
<td>0.093</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>Shallow cavity</td>
<td>open pipe</td>
<td>0.115</td>
<td>0.077</td>
<td>0.375</td>
<td>0.093</td>
<td>9</td>
</tr>
</tbody>
</table>
unstable grazing flows above cavities.\textsuperscript{13} This phenomenon is illustrated in Fig. 9 for two arrangements of Table I, one associated with fixed boundary conditions and a variable flow velocity, and one associated with a constant flow velocity and variable boundary conditions.

In the case of Fig. 9(a), a reverberating expansion chamber is arranged upstream, a perforated pipe with one foam disk is arranged downstream, and the flow velocity varies from 6 to 12 m s\textsuperscript{-1}. Two range of frequencies around 1500 and 2100 Hz are observed for velocities below and above 8 m s\textsuperscript{-1}, respectively. The frequency step is the signature of the shift from one acoustic mode to another when the flow velocity is increased. In the case of Fig. 9(b), the flow velocity is equal to 9 m s\textsuperscript{-1}, and the shallow cavity depth $d_{cav}$ varies from 0.03 to 0.06 m. Two ranges of frequencies around 2100 and 1800 Hz are observed for cavity depths below and above 0.04 m, respectively. The frequency step is due to the shift from one acoustic mode to another, under the influence of the reflection coefficient evolution of the cavity as illustrated in Fig. 6. In both cases, the whistling frequency corresponds to an acoustic mode of the orifice and of the surrounding pipe, associated with a value of the Strouhal number in the range 0.2–0.4, consistent with the acoustic amplification range in Fig. 1.

More precisely, the first acoustic mode in Fig. 9(a) is associated with a Strouhal number varying from 0.32 to 0.27, and the second acoustic mode is associated with a Strouhal number varying from 0.32 to 0.23, whereas the first acoustic mode in Fig. 9(b) is associated with a Strouhal number equal to 0.29, and the second acoustic mode is associated with a Strouhal number equal to about 0.25. In the first case, the full range of Strouhal numbers prone to whistling is covered, whereas in the second case, discrete values of the Strouhal number and of the whistling frequency are observed.

A better understanding of the whistling frequency values can be obtained with the help of the linear stability analysis introduced in Sec. II C. The gray triangles of Fig. 9 label the unstable frequencies derived from the linear acoustic gain of Fig. 1 and the boundary conditions provided in Sec. III B. The unstable frequencies predicted by the linear analysis fairly agree with the measured whistling frequencies, which constitutes a new result in scientific literature; to the best of the authors' knowledge, such a procedure has not been applied to whistling orifices up to now. For practical purposes, it can be assessed that self-sustained oscillations occur at a frequency such that the product $GF$ is a real number, with a value higher than unity. Whistling can hence appear if the feedback term $F$ has a modulus typically higher than 0.5, and if the frequency corresponds to a Strouhal number in the range 0.2–0.4.

It is worth mentioning the fact that, in the previous discussion, it was understated that the unstable frequency is very close to the actual whistling frequency. Such was the case in all experiments performed in the present study, and although the two frequencies can theoretically be different, as pointed out by Sarpkaya,\textsuperscript{32} and as the evolution of the argument of $G_{sat}$ illustrated in Fig. 3 suggests, it was not possible to obtain experimental evidence of the difference of these two frequencies. Further work is needed to investigate this alleged difference.

At last, as illustrated in Fig. 9(b), two unstable frequencies can be predicted by the linear stability analysis, whereas whistling occurs over a single frequency. The other way around, two whistling frequencies were sometimes observed during experiments. Such situations are normal, for two natural frequencies can be present in the Strouhal range 0.2–0.4. Further work is also needed to investigate the stability of a lock-in regime in this case, and to determine if selection mechanisms do exist when two unstable frequencies are present.

### B. Whistling amplitude

The next step of the study takes into account nonlinear effects, and the goal is now to study the evolution of the whistling amplitude with the acoustic feedback $F$ and with other nondimensional parameters. In order to highlight the salient features of the whistling phenomenon, the discussion is first focused on whistling occurring on a single acoustic mode. Particularly, the analysis is limited to whistling at about 2100 Hz for the arrangements numbered 2–4, as observed in Fig. 9(a). As defined in Table I and illustrated in

![Fig. 9. Comparison between the calculated unstable frequencies (gray triangles) and the measured whistling frequencies (white squares). The upper plot (a) stands for arrangement 3, where the boundary conditions are constant and where the flow velocity varies from 6 to 12 m s\textsuperscript{-1}. The lower plot (b) stands for arrangement 9, where the flow velocity is kept equal to 9 m s\textsuperscript{-1} and where the upstream boundary condition is tuned by adjusting the depth of the shallow cavity.](image-url)
Fig. 5, these three arrangements have similar phase conditions and only the amplitude of the downstream reflection changes between each arrangement. In the meanwhile, the modulus of the reflection coefficient varies from 0.5 to 0.8, so that the acoustic modes of the system are only slightly altered between each arrangement.

Figure 10 illustrates the main result of the study: The parameters controlling the whistling amplitude are the modulus of the acoustic feedback $\mathcal{F}$ and the Strouhal number. The dimensionless acoustic velocity varies from 1% to about 15%, it increases with the feedback modulus [see Fig. 10(b)], and it exhibits a maximum for a Strouhal number of the order of 0.25 [see Fig. 10(a)]. It can also be observed that the highest amplitude is obtained for a Strouhal number close to the value which makes the linear gain maximum [see Fig. 1(a)]. Moreover, the comparison of the three curves of Fig. 10(a) indicates a variation of the Strouhal number associated with the maximum amplitude with the feedback modulus. However, this variation is within the range of uncertainties of the measurement procedure, and it is not clear whether other parameters like the argument of $\mathcal{F}$ should not also be taken into account. Due to the lack of very accurate data, this question is not investigated in the present study.

The previous analysis is now extended to all whistling regimes observed during the experiments summarized in Table I. Notwithstanding the upstream or downstream arrangements or the flow velocity, data are sorted in Fig. 11 by the values of the Strouhal number and of the modulus of the feedback only. Whistling regimes exhibiting values of the modulus of $\mathcal{F}$ equal to 0.56, 0.65, 0.71, and 0.78 are selected, so that the influence of this parameter is made apparent. Other data are available that are consistent with the ones provided hereafter, but with intermediate values of the modulus of $\mathcal{F}$. They are not plotted for the sake of clarity. A trend very similar to the one of the single mode case is observed. The dimensionless acoustic velocity varies from 0.5% to about 12%, it increases with the feedback modulus [see Fig. 11(b)], and it exhibits a maximum for a Strouhal number of the order of 0.25 [see Fig. 11(a)]. These data are consistent with the ones of Fig. 10, but they exhibit more scatter, which is due to the fact that they were obtained with different acoustic boundaries, so that they do not correspond to one single acoustic mode. To the authors’ knowledge, the dependence of the whistling amplitude to the Strouhal number and to the modulus of the acoustic feedback coefficient is a new result. It is suggested that future works in the field of orifice whistling explicitly indicate the values of these two parameters.
C. Discussion

Based on the results of Sec. IV B and on a further analysis of the tests used in Fig. 10, the mechanisms of acoustic amplification and saturation are discussed here.

First, the saturated gain can be estimated for the studied tests with the help of Eq. (11). Using the values of the inverse of the feedback modulus in Fig. 10, the modulus of $G_{cut}$ is found to be equal to about 1.6 (white squares), to 1.4 (light gray squares), and to 1.2 (dark gray squares). As expected, the saturated gain exceeds unity, and it is lower than the linear gain in Fig. 1(a). Its decrease with the increase of the whistling amplitude is an illustration of the adaptation of vortex shedding to the acoustics of the duct.

Considering the saturated gain of the orifice as a function of the Strouhal number and of the acoustic velocity, as proposed in Sec. II, a qualitative description of the initial instability and of its evolution toward the steady state regime can be given. At the onset of instability, the linear gain of the orifice shown in Fig. 1 exceeds the feedback modulus in a certain range of frequencies. If one of the frequencies is such that the argument of $F$ equals the argument of $G$, lock-in occurs and an exponentially growing velocity is generated at this frequency. When the amplitude of the fluctuating velocity reaches a value of a few percent of the steady flow velocity, nonlinear effects reduce the modulus of the acoustic gain and alter its argument simultaneously. Hence, both the whistling amplitude, and the whistling frequency appear as consequences of a nonlinear adaptation of the orifice acoustic gain to its surroundings.

Next, the issue of the evolution of the whistling frequency with the flow velocity can be raised. For a given mode, a slight increase of the frequency is observed, in a manner similar to side branches cases.26,29 A variation of the flow velocity requires an adaptation of the gain of the orifice. It can be gathered that the increase of the flow velocity results in a decrease of the Strouhal number, which in turn generates an increase of the argument of the linear gain $G$ as illustrated in Fig. 1(b). The instability frequency, as well as the whistling frequency, varies in such a way that the argument of $G$ balances the argument of $F$. To support this view, the evolution of the argument of $F$ as a function of the flow velocity is plotted in Fig. 12(b) for the whistling mode of tests 2–4, which appears at 2100 Hz as plotted in Fig. 12(a).

The first result of the study is the successful application of the Bode–Nyquist criterion to the definition of the onset of self-sustained oscillations; it is found that whistling can occur at a frequency such that $GF$ is a real number, with a value higher than one. The unstable frequency obtained with this procedure is very close to the whistling frequency. For practical purposes, this criterion demonstrates that whistling occurs only for frequencies such that the modulus of the feedback coefficient $F$ exceeds 0.5, and for Strouhal numbers in the range 0.2–0.4.

The second result of the study is the fact that the steady whistling regime results from the adaptation of the orifice flow dynamics to the acoustic modes of the surrounding duct. The experiments have shown that the acoustic velocity generated by the orifice varies from 0.01 to 0.15 times the steady flow velocity. This velocity exhibits a maximum for a Strouhal number of the order of 0.25, and it increases with the modulus of $F$.

Further work is needed to generalize the results of the present study to other orifices, and to investigate the influence of the Mach number and of the Reynolds number. Application of the procedure to other devices like Anderson’s orifice pipe termination9 could also be considered. Attention could be given to the issue of orifice whistling at two different frequencies, when several natural frequencies...
are in the range of Strouhal number enhancing self-sustained oscillations. More specifically, the selection mechanisms of one unstable frequency and the stability of the steady state regime should be investigated.


