Digital holographic reconstruction of large objects using a convolution approach and adjustable magnification

Jun-chang Li,[1,2,3] Patrice Tankam,[2,4] Zu-jie Peng,[1,5] and Pascal Picart[2,*]

[1]*Kunming University of Science and Technology, Kunming 650093, China*
[2]*LAUM, CNRS, Université du Maine, Avenue Olivier Messiaen, 72085 Le Mans, France*
[3]*jcli@vip.163.com*
[4]*patrice.tankam.etu@univ-lemans.fr*
[5]*pengzujie@163.com*
**Corresponding author: pascal.picart@univ-lemans.fr*

Received November 10, 2008; revised December 27, 2008; accepted January 6, 2009; posted January 15, 2009 (Doc. ID 103889); published February 19, 2009

We present a numerical method for reconstructing large objects using a convolution method with an adjustable magnification. The method is based on the image locations and magnification relations of holography when the illuminating beam is a spherical wavefront. A modified version of the angular spectrum transfer function is proposed that allows the filtering in the spatial frequency spectrum. Experimental results confirm the suitability of the proposed method. © 2009 Optical Society of America

OCIS codes: 090.0090, 090.1995, 090.1760, 100.2000, 100.3010.

In digital holography the numerical reconstruction of the object encoded using optical interferences is usually based on the discrete version of the Fresnel transform, on the computation of the convolution formula of diffraction [1–3], or on the angular spectrum method [4,5]. The discrete Fresnel transform is very well adapted to large objects, i.e., objects with lateral dimensions quite greater than that of the recording area. With this method, the sampling pitch in the reconstructed plane depends on the wavelength of the light used for the recording, on the diffraction distance, and on the number of data points used for the computation [1–3]. However, such an algorithm is not suitable for digital color holography, except if one uses a zero-padding adapted to the wavelength [6]. Objects with dimensions on the order of the recording area can be reconstructed by either the Fresnel transform or the convolution approach. With convolution, the field of view conserves the same physical dimension as that of the recording. To extend the size of the reconstructed field, Kreis *et al.* [1] and Yamaguchi *et al.* [7] used a zero-padding of the algorithm in the convolution method. This approach can be intuitively thought, since the reconstruction horizon obtained by the convolution approach is related to the one of the sensors. So increasing the size of the reconstruction horizon should increase the size of the reconstructed field and then the size of the object. It follows that the necessary number of data points is simply given by the ratio $L=\Delta A_x/p_x$ (for the x direction, for example, p_x, pixel pitch of the sensor; ΔA_x, size of the object in the x direction, similar relation in the y direction). For example, if $\Delta A_x=\Delta A_y=60$ mm and $p_x=p_y=5\ \mu$m, the algorithm needs $(L,K)=(12{,}000,12{,}000)$ data points. However, the computation time and the memory used for such processing may exceed the one of usual personal computers. In 2004, Zhang *et al.* [8] proposed an algorithm based on a double Fresnel transform allowing the adjustment of the side length of the field of view. Demonstration was performed with an object with moderated size with a field of view of 10 mm × 10 mm. This Letter proposes an alternative algorithm based on the convolution approach, needing only two fast Fourier transform computations and allowing the reconstruction of a large object size variety. The principle of the algorithm is a fruitful mixing between the zero-padding strategy and the fundamental properties of holography. Consider the conjugation relations of holography established when the reconstructing wave is a spherical wavefront [9]. Note that Yamaguchi *et al.* theoretically described the use of a spherical wave as an illuminating wave but convenient algorithm, and experimental results were not described [10]. The reconstructing wavefront can be in the form of a Rayleigh–Sommerfeld spherical wave described in Eq. (1) (for the coordinate chart, see Fig. 1):

$$w(x,y,R_c)=\frac{iR_c}{\lambda}\frac{\exp[2i\pi\,\mathrm{sgn}(R_c)/\lambda\sqrt{R_c^2+x^2+y^2}]}{R_c^2+x^2+y^2}, \quad (1)$$

where sgn is the sign function, $i=\sqrt{-1}$, R_c is the curvature radius, and λ is the wavelength. The conjuga-

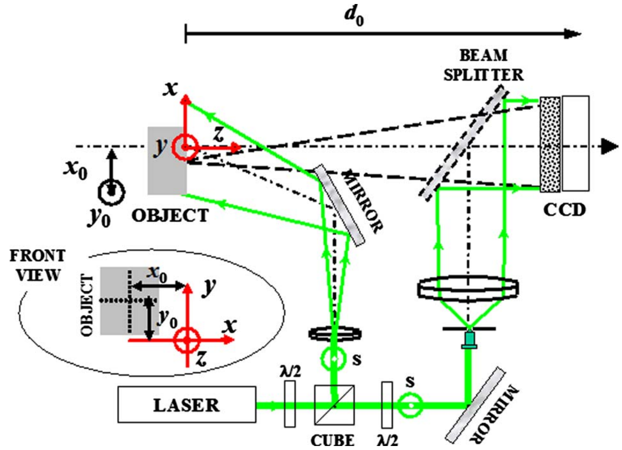

Fig. 1. (Color online) Experimental setup ($d_0=1500$ mm, $x_0=53.6$ mm, $y_0=-51.9$ mm).

tion relations of holography ([9], p. 316) indicate that the position of the reconstructed object is given by $1/d_R = -1/d_0 + 1/R_c$, where d_0 is the distance between the recording area and the object ($d_0 > 0$). Furthermore the transversal magnification between the reconstructed object and the real one is given by $\gamma = -d_R/d_0$. Since the convolution approach imposes the size of the reconstructed horizon, consider that it is (Lp_x, Kp_y). The transversal magnification γ must be chosen according to the ratio $Lp_x/\Delta A_x$ (or $Kp_y/\Delta A_y$): if greater than $Lp_x/\Delta A_x$ then the object will not fully lie in the reconstructed area, if smaller than $Lp_x/\Delta A_x$, the object will be fully included in the field of view (and, respectively, for y). Consequently, γ sets the useful values for the reconstructing distance d_R and the curvature radius of the spherical wavefront according to $R_c = \gamma d_0/(\gamma - 1)$. In a convolution strategy, the change in the reconstructing distance ($d_R \neq -d_0$) modifies the useful spatial frequency bandwidth of the transfer function, which is then given by $(\Delta u, \Delta v) = (Lp_x/\lambda d_R, Kp_y/\lambda d_R)$ [11]. The transfer function for the numerical reconstruction can be chosen to be the angular spectrum transfer function [3,4,9] for which the Shannon theorem requires a sampling pitch along the frequency axis given by (a similar relation holds in the perpendicular direction for δv)

$$\delta u \leqslant \frac{\sqrt{1 - \lambda^2 \Delta u^2/4}}{\lambda d_R \Delta u} \cong \frac{1}{\lambda d_R \Delta u}. \quad (2)$$

Since the spatial frequency spectrum is computed with $K \times L$ data points, the Shannon theorem is fulfilled in the useful bandwidth, because the FFT computation is automatically setting $(\delta u, \delta v) = (1/Lp_x, 1/Kp_y)$. This also means that the Shannon theorem is not fulfilled for the full spatial bandwidth $(1/p_x \times 1/p_y)$. Furthermore, in digital off-axis holography, the reference wave includes a spatial biased phase whose spatial frequencies are (u_0, v_0) and that localizes the object in the reconstructed space and in the Fourier space. Thus the transfer function must be adapted in order to first fulfill the Shannon theorem in the frequency space and last to take into account that the object spectrum is localized at (u_0, v_0). So we chose for the transfer function a modified version of the angular spectrum transfer function according to the following equation:

$$\tilde{H}(u,v,d_R) = \begin{cases} \exp[2i\pi d_R/\lambda \sqrt{1 - \lambda^2(u-u_0)^2 - \lambda^2(v-v_0)^2}], & \text{if } |u-u_0| \leqslant Lp_x/\lambda d_R,\ |v-v_0| \leqslant Kp_y/\lambda d_R \\ 0, & \text{elsewhere} \end{cases}. \quad (3)$$

The use of a spherical wave as an illuminating wave modifies the width of the zero-order diffraction, which is then $(Np_x/\lambda R_c, Mp_y/\lambda R_c)$. Ideally, the algorithm must work with values of the parameters such that there is no overlapping between the useful bandwidth and the zero-order bandwidth. If there is overlapping, the reconstructed field will include a part of the parasitic zero-order diffraction. A sufficient condition to avoid overlapping is given by

$$\frac{Np_x}{2\lambda |R_c|} + \frac{Lp_x}{2\lambda |d_R|} < |u_0|. \quad (4)$$

A similar relation holds for the vertical direction. In this Letter, the aim is to reconstruct objects with a size larger than the recording area. Thus the magnification must be less than 1 and must be chosen positive if one wants to see the object sitting up straight. This leads to $|\gamma - 1| = 1 - \gamma$ and, from Eq. (4), Eq. (5) gives the boundary values for the magnification,

$$\frac{(L+N)p_x}{2\lambda d_0 |u_0| + Np_x} < \gamma < \frac{Lp_x}{\Delta A_x}. \quad (5)$$

A similar relation holds for the vertical direction.

To illustrate the potentialities of this strategy, consider the optical setup described in Fig. 1, which is based on a Mach–Zehnder interferometer, a continuous green laser ($\lambda = 532$ nm), and a CCD camera (PCO Pixel Fly, $(M,N) = (1024, 1360)$ pixels with pitches $p_x = p_y = 4.65$ μm). The object under consideration is the bronze medal of the half marathon "20 km de Paris 2000," which is 60 mm in diameter. The distance for the recording is set to $d_0 = 1500$ mm, and the object is translated perpendicularly to the optical axis of quantities ($x_0 = 53.6$ mm, $y_0 = -51.9$ mm) thus producing off-axis spatial frequencies $(u_0, v_0) = (x_0/\lambda d_0, y_0/\lambda d_0) \approx (67.2\ \text{mm}^{-1}, -65.1\ \text{mm}^{-1})$ [11]. As indicated previously, the magnification depends on the size of the reconstructed area. If the reconstructing area is the same as the recording area then $(K,L) = (M,N)$, the magnification must be chosen to be $\gamma = 0.0794$, the curvature radius to be $R_c = -128.66$ mm, and the reconstruction distance must be set to $d_R = -118.5$ mm. However Eq. (5) leads to $0.0976 < \gamma < 0.0794$, which is incompatible, so this means that the reconstructed field will be perturbed by the zero-order diffraction. Consequently, a zero-order free reconstruction with $(K,L) = (M,N)$ is impossible. By using zero-padding to extend the reconstruction horizon to $(K,L) = (2048, 2048) \gg (M,N)$ data points corresponding to about twice that of the recording sensor, Eq. (5) leads to $0.1395 < \gamma < 0.1587$. So upper choice $\gamma = 0.158$ appears to be a good compromise. This leads to curvature radius $R_c = -281.47$ mm and reconstruction distance $d_R = -237$ mm. Note that lower choice $\gamma = 0.140$ is also possible, leading to $R_c = -244.18$ mm,

$d_R = -210$ mm, and the object will be slightly smaller than the reconstructed area but there will be no perturbation in the field of view. If one chooses a smaller value than $\gamma = 0.1395$, then the perturbation due to the zero order will appear in the field of view. For example, with $\gamma = 0.118$, $R_c = -201.64$ mm and $d_R = -171.75$ mm, the reconstructed object is smaller than the reconstruction horizon, and perturbation will lie in the field of view. Figure 2 shows the spatial frequency spectrum obtained for the hologram multiplied by the spherical wavefront, for the four cases presented above named cases a, b, c, d, corresponding to $\gamma = \{0.079; 0.158; 0.140; 0.118\}$. The white square line corresponds to the useful bandwidth of the angular spectrum transfer function, and the white cross is localized at spectral coordinate (u_0, v_0). The useful bandwidth occupies $\Delta u/\delta u \times \Delta v/\delta v = L^2 p_x{}^2/\lambda d_R \times K^2 p_y{}^2/\lambda d_R = \{634, 719, 811, 992\} \times \{359, 719, 811, 992\}$ data points in the spectrum for cases $\{a, b, c, d\}$, respectively. Figure 3 shows the object reconstructed with the parameters of the four cases a, b, c, d. For cases b and c the object is fully lying in the reconstructed area, thus validating the proposed approach. The computation time is 3.7 s for $K = 1024$, $L = 1360$, and it is 9.2 s for $K = L = 2048$ with MATLAB5.3 on a computer equipped with a Pentium 2.33 GHz processor and 2 GoRAM. Note that the choice of γ does not influence the intrinsic resolution of the reconstructed image: indeed the method uses diffraction calculation mixed with zero-padding and, as mentioned in [11], the intrinsic image resolution depends only on the recording conditions. As discussed, a perturbation is included in cases a and d, which is provided by the zero-order diffraction. Indeed, the spatial bandwidth due to the spherical wave is more extended than in cases a, d, because the curvature radius is decreased and Eq. (4) is not fulfilled. Figure 2 clearly shows overlapping between useful and parasitic bandwidth. Whereas in cases b, c, the filtering window around spatial frequencies (u_0, v_0) [Eq. (3)] does not include any contribution of the zero-order diffraction; thus Eq. (4) is fulfilled.

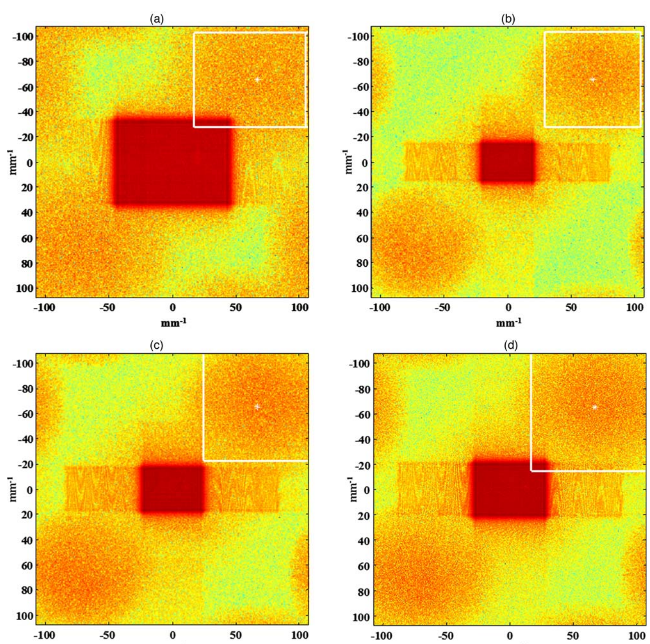

Fig. 2. (Color online) Spatial frequency spectrum for (a) $\gamma = 0.079$, (b) $\gamma = 0.158$, (c) $\gamma = 0.140$, and (d) $\gamma = 0.118$.

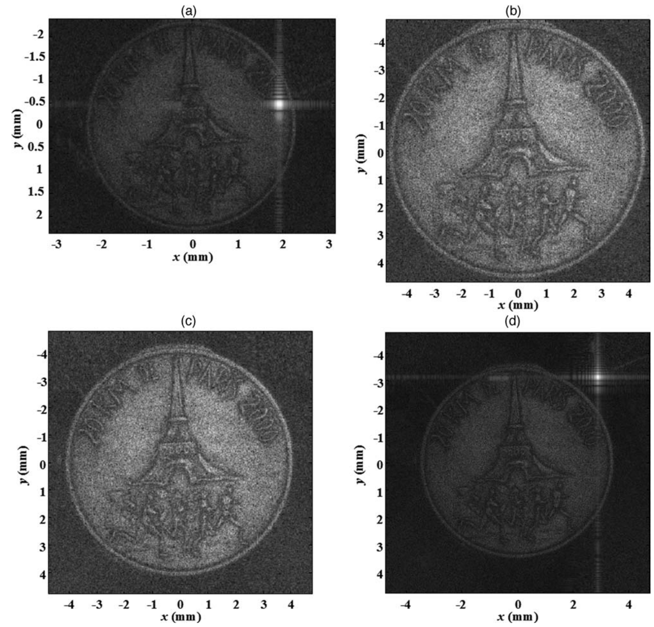

Fig. 3. Reconstructed field of view for (a) $\gamma = 0.079$, $K = M = 1024$, $L = N = 1360$, (b) $\gamma = 0.158$, $K = L = 2048$, (c) $\gamma = 0.140$, $K = L = 2048$, and (d) $\gamma = 0.118$, $K = L = 2048$.

In conclusion, this Letter presents a numerical method for reconstructing a large object using a convolution method with an adjustable magnification. The method is based on the image locations and magnification relations of holography and on a spectral filtering in the Fourier space. The filtering is realized using a modified version of the angular spectrum transfer function according to the necessary useful bandwidth for the reconstruction horizon. Experimental results show very good agreement with the theoretical analysis.

References

1. Th. Kreis, M. Adams, and W. Jüptner, Proc. SPIE **3098**, 224 (1997).
2. D. Mas, J. Perez, C. Hernandez, C. Vazquez, J. J. Miret, and C. Illueca, Opt. Commun. **227**, 245 (2003).
3. J. C. Li, Z. Peng, and Y. Fu, Opt. Commun. **280**, 243 (2007).
4. L. Yu and M. K. Kim, Opt. Lett. **30**, 2092 (2005).
5. L. Yu and M. K. Kim, Opt. Lett. **31**, 897 (2006).
6. P. Ferraro, S. De Nicola, G. Coppola, A. Finizio, D. Alfieri, and G. Pierattini, Opt. Lett. **29**, 854 (2004).
7. I. Yamaguchi, T. Matsumura, and J. Kato, Opt. Lett. **27**, 1108 (2002).
8. F. Zhang, I. Yamaguchi, and L. P. Yaroslavsky, Opt. Lett. **29**, 1668 (2004).
9. J. W. Goodman, *Introduction to Fourier Optics*, 2nd ed. (McGraw-Hill, 1996).
10. I. Yamaguchi, J. Kato, S. Ohta, and J. Mizuno, Appl. Opt. **40**, 6177 (2001).
11. P. Picart and J. Leval, J. Opt. Soc. Am. A **25**, 1744 (2008).